Next Generation Higher National Unit Specification

Engineering Mathematics 4 (SCQF level 8)

Unit code: J7LA 48

SCQF level: 8 (8 SCQF credit points)

Valid from: session 2024 to 25

Prototype unit specification for use in pilot delivery only (version 1.0) February 2024

This unit specification provides detailed information about the unit to ensure consistent and transparent assessment year on year.

This unit specification is for teachers and lecturers and contains all the mandatory information required to deliver and assess the unit.

This edition: February 2024 (version 1.0)

Unit purpose

This unit develops the mathematical skills learners need to progress from the Higher National Diploma (HND) to degree study. It provides opportunities for learners to develop the knowledge, understanding and skills to solve problems involving the use of complex algebra, matrices and mathematical series. Learners also learn techniques to solve first-order differential equations.

The target learner group for the unit is learners who want to further develop their knowledge of mathematics to support a career in engineering fields such as:

- electrical engineering
- mechanical engineering
- systems engineering
- manufacturing engineering
- measurement and control engineering

It is also aimed at learners who want to develop the practical, personal and professional skills required for a successful career as an engineering technician.

Entry to the unit is at your centre's discretion. However, we recommend that learners have a good knowledge and understanding of differential and integral calculus, together with sound numerical and algebraic skills. This may be evidenced by having completed the SCQF level 7 units Engineering Mathematics 2 and Engineering Mathematics 3.

Unit outcomes

Learners who complete this unit can:

1 solve problems involving complex algebra
2 solve problems involving the manipulation of matrices
3 solve problems requiring the use of series representation of functions
4 solve first-order differential equations

Evidence requirements

You should use a sampling approach to assess knowledge and skills. Learners must provide written or recorded oral evidence to demonstrate their knowledge and skills across all outcomes. Where evidence for outcomes is assessed on a sample basis, you must teach the whole of the content listed in the 'Knowledge and skills' section and make it available for assessment.

We recommend you assess all four outcomes using a single end-of-unit assessment. You must use different assessment instruments, and a different sample of knowledge and skills, for re-assessments.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions. Learners must not use computer algebra in the assessment of this unit.

To successfully achieve the unit, learners must provide written and/or oral recorded evidence that they have completed the following outcomes.

Outcome 1

Sample any two of the three required items:

- Evaluate one problem involving the multiplication of two complex numbers and one problem involving the division of two complex numbers, and represent the results on an Argand diagram.
- Solve one problem involving the use of the Euler's formula (for example, converting a complex number or expression into exponential form, or vice versa, representing a trigonometrical function in exponential form).
- Solve one problem using De Moivre's theorem (for example, raising a complex number to the n^{th} power, proving a trigonometrical identity).

Outcome 2

Sample any three of the five required items:

- Define the meaning of the term ‘matrix’ in the context of mathematics (this may include special cases such as square, identity, transpose).
- Solve one problem involving the scalar multiplication, addition or subtraction of two matrices and one problem involving the multiplication of two matrices.
- Find the determinant of a 3×3 matrix.
- Find the inverse of a 3×3 matrix.
- Apply Gaussian elimination to solve a system of linear equations containing three variables.

Outcome 3

Sample any two of the four required items:

- Derive the Taylor series of a given function.
- Derive the Maclaurin series of a given function.
- Expand the product of two functions using series forms.
- Develop a linear approximation of one polynomial or one non-linear function.

Outcome 4

Sample any two of the four required items:

- Solve a first-order differential equation using the method of separation of variables.
- Solve a first-order differential equation by substitution.
- Solve a first-order differential equation using the integrating factor method.
- Solve a first-order differential equation with an initial value using one of the three methods shown above.

Knowledge and skills

The following table shows the knowledge and skills covered by the unit outcomes:

Knowledge	Skills
Outcome 1 Learners should understand: ♦ complex algebra and Argand diagrams ♦ Euler's formula ♦ De Moivre's theorem	**Outcome 1** Learners can: ♦ evaluate one problem involving the multiplication of two complex numbers and one problem involving the division of two complex numbers and represent the results on an Argand diagram ♦ solve one problem involving the use of the Euler's formula (for example, converting a complex number or expression into exponential form, or vice versa, representing a trigonometrical function in exponential form) ♦ solve one problem using De Moivre's theorem (for example, raising a complex number to the n^{th} power, proving a trigonometrical identity)
Outcome 2 Learners should understand: ♦ definitions of 'matrix' ♦ matrix operations ♦ determinants ♦ inverse matrices ♦ Gaussian elimination	**Outcome 2** Learners can: ♦ define the meaning of the term 'matrix' in the context of mathematics (this may include special cases such as square, identity, transpose) ♦ solve one problem involving the scalar multiplication, addition or subtraction of two matrices, and one problem involving the multiplication of two matrices ♦ find the determinant of a 3×3 matrix ♦ find the inverse of a 3×3 matrix ♦ apply Gaussian elimination to solve a system of linear equations containing three variables

Knowledge	Skills
Outcome 3 Learners should understand: ♦ Maclaurin series ♦ Taylor series ♦ expansion of products of functions using series forms ♦ linear approximations of polynomial and non-linear functions	**Outcome 3** Learners can: ♦ derive the Taylor series of a given function ♦ derive the Maclaurin series of a given function ♦ expand the product of two functions using series forms ♦ develop a linear approximation of one polynomial or one non-linear function
Outcome 4 Learners should understand: ♦ separation of variables for first-order differential equations ♦ substitution for first-order differential equations ♦ integrating factor method for first-order differential equations ♦ initial value problems using the above methods for first-order differential equations	**Outcome 4** Learners can: ♦ solve a first-order differential equation using the method of separation of variables ♦ solve a first-order differential equation by substitution ♦ solve a first-order differential equation using the integrating factor method ♦ solve a first-order differential equation with an initial value using one of the three methods shown above

Meta-skills

Throughout the unit, learners develop meta-skills to enhance their employability in the engineering sector.

Self-management

Learners develop the skills of focusing, adapting and initiative as they solve engineering problems.

Social intelligence

Learners develop the meta-skill of communicating as they ask questions and receive information from lecturers.

Innovation

Learners develop the meta-skills of curiosity, sense-making and critical thinking as they apply mathematical techniques to problem solving.

Literacies

Numeracy

Learners develop their numeracy skills by solving engineering problems using mathematics.

Communication

Learners develop their communication skills by studying the course material, and engaging with other learners, and their teachers or lecturers.

Digital

Learners develop digital literacy by accessing the course material through a virtual learning environment (VLE) and using software tools to solve engineering problems.

Delivery of unit

This unit provides the core mathematical principles and processes that underpin Higher National Certificate (HNC) and HND Engineering units across a range of disciplines. We recommend you deliver the unit towards the beginning of these awards.

The unit is one of a suite of five units in mathematics developed for Higher National Qualifications across a range of engineering disciplines. The five units are:

- Engineering Mathematics 1 at SCQF level 6
- Engineering Mathematics 2 at SCQF level 7
- Engineering Mathematics 3 at SCQF level 7
- Engineering Mathematics 4 at SCQF level 8
- Engineering Mathematics 5 at SCQF level 8

The amount of time you allocate to this unit is at your centre's discretion. However, the notional design length is 40 hours.

The amount of time you allocate to each outcome is also at your discretion. We suggest the following distribution of time, including assessment:

Outcome 1 — Solve problems involving complex algebra
(10 hours)

Outcome 2 — Solve problems involving the manipulation of matrices
(10 hours)

Outcome 3 — Solve problems requiring the use of series representation of functions
(5 hours)

Outcome 4 — Solve first-order differential equations
(15 hours)

Additional guidance

The guidance in this section is not mandatory.

Content and context for this unit

We strongly recommend you use the following list of topics to ensure continuity of learning and teaching.

Solve problems involving complex algebra (outcome 1)

- Introduce j in terms of $\mathrm{j}^2 = -1,\ \mathrm{j} = \sqrt{-1}$
- Complex and polar forms.
- Representation of complex numbers on an Argand diagram. Comment on the domain of the principal argument, $\arg(z)$
- Addition and subtraction of complex numbers.
- Multiplication of complex numbers in rectangular and polar forms.
- Complex conjugates.
- Division of complex numbers in rectangular and polar form.
- Introduce Euler's formula: $\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + \mathrm{j}\sin\theta$ or $\mathrm{e}^{-\mathrm{j}\theta} = \cos\theta - \mathrm{j}\sin\theta$
  $z = r(\cos + \mathrm{j}\sin\theta) = re^{\mathrm{j}\theta}$ and $z = r(\cos\theta - \mathrm{j}\sin\theta) = re^{-\mathrm{j}\theta}$
- Solve problems involving the use of Euler's formula. For example, express the following in exponential form: $-10, (-2 + \mathrm{j}5), (1 - \mathrm{j})(1 + 3\,\mathrm{j})$ given that $z_1 = 6\mathrm{e}^{\mathrm{j}\pi/6}$ and $z_2 = 9\mathrm{e}^{-\mathrm{j}\pi/3}$ determine $z_1 z_2$ and z_1 / z_2
- Introduce De Moivre's theorem as $(\cos\theta + \mathrm{j}\sin\theta)^n = \cos n\theta + \mathrm{j}\sin n\theta$
  $z^n = r^n \angle n\theta$
- Solve problems involving De Moivre's theorem. For example, express $\left(\sqrt{2} + \mathrm{j}\right)^5$ in the form $a + \mathrm{j}b$, use De Moivre's theorem to express $\sin 3\theta$ as powers of $\sin\theta$
- Solve complex equations. For example, $z^3 + 7z = 0$, $z^2 + 2z + 2 = 0$, $z^3 - 8 = 0$

Solve problems involving the manipulation of matrices (outcome 2)

- Define what is meant by the term 'matrix' in the context of mathematics.
- Identify some special matrices (for example, square, identity, transpose, augmented).
- Multiply a matrix by a scalar.
- Add and subtract matrices.
- State the conditions for multiplying matrices.
- Multiply matrices.
- Explain the technique for calculating the determinant of a 2 × 2 matrix.
- Explain what is meant by the minor and the cofactor of an element.
- Explain the technique for calculating the determinant of a 3 × 3 matrix.

- Calculate the determinants of 3 × 3 matrices.
- Explain the technique for calculating the determinant of a 4 × 4 matrix to demonstrate the extension of the method to higher-order matrices (if time permits).
- Calculate the determinants of simple 4 × 4 matrices (if time permits).
- Explain the way in which to find the inverse of a 2 × 2 matrix using the determinant and cofactor method.
- Determine the inverses of 2 × 2 matrices using the determinant and cofactor method.
- Explain the determinant and cofactor method for finding the inverse of a 3 × 3 matrix.
- Determine the inverses of 3 × 3 matrices using the determinant and cofactor method.
- Explain the Gaussian elimination technique for finding the inverse of any matrix and apply it to examples up to and including echelon form.
- Use Gaussian elimination techniques to solve engineering-related problems.

Solve problems requiring the use of series representation of functions (outcome 3)

- Distinguish between a sequence and a series.
- Brief treatment of arithmetic and geometrical sequences.
- Introduce Taylor's series as:

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + (x - x_0)^2 \frac{f''(x_0)}{2!} + \ldots + (x - x_0)^n \frac{f^{(n)}(x_0)}{n!} + \ldots$$

- Determine the series representation of functions using Taylor's series around a variety of x_0 values.
- Introduce the series representation of functions using Maclaurin's series:

$$f(x) = f(0) + xf'(0) + x^2 \frac{f''(0)}{2!} + \ldots + x^n \frac{f^{(n)}(0)}{n!} + \ldots$$

- Expand the product of two functions using series forms. For example, $e^x \ln(1+x), e^x \tan x$. You could provide learners with a table showing the series expansion of standard functions such as $e^x, \sin x, \cos x, (1+x)^n$
- Develop a linear approximation of one polynomial or one non-linear function. For example, $\sin x, \cos x, e^{-x}$

Solve first-order differential equations (outcome 4)

- Introduction to differential equations — explain the meaning of dependent and independent variables, order of a differential equation, linearity, initial conditions, and so on.
- Solve first-order differential equations by direct integration.
- Introduce the separation of variables method as applying to equations of the following form:

$$\frac{dy}{dx} = f(x) g(y)$$

- Apply the method of separation of variables to solve first-order differential equations (including first-order differential equations that arise in engineering).
- Introduce the method of solving differential equations by substitution. For example, using $y = vx$ or $y = ax + by + c$
- Apply the method of substitution to the solution of first-order differential equations.
- Introduce the integrating factor method as applying to first-order differential equations of the following form:

  $$\frac{dy}{dx} + p(x)y = q(x)$$

  where the integrating factor $e^{\int p(x)dx}$

- Apply the integrating factor method to the solution of first-order differential equations.
- Illustrate how some first-order differential equations may be solved by more than one method.
- Solve first-order differential equations taken from engineering with an initial value using one of the methods shown above.

Approaches to delivery

The unit provides many of the core mathematical principles and processes learners need to study engineering at a more advanced level. Given the subject matter, we recommend you deliver this unit after learners have studied Engineering Mathematics 3.

You can deliver the outcomes in any order.

We recommend that you deliver the unit using a mainly didactic approach. You should supplement teaching with formative assessment where learners have opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the unit. If appropriate, you could deliver the unit as a series of lectures supported by tutorial sessions to help learners prepare for degree-level study.

You can use computer software and computer algebra to support learning (for example, to confirm the solutions of mathematical problems), but we strongly recommend that these learning resources are only used in a supportive capacity and not as the principal means of delivering unit content.

Approaches to assessment

We recommend using an examination question paper. This should comprise an appropriate balance of short-answer, restricted-response and structured questions. You should not group or label questions by outcome.

Learners should not see the assessment paper before the assessment takes place. You must ensure the security, integrity and confidentiality of the assessment papers at all times. You should conduct assessments under closed-book, controlled and invigilated conditions.

The total assessment time for the four outcomes should not exceed two hours. When you assess learners' responses in a summative assessment, you should principally concentrate on their ability to apply the correct mathematical technique and processes. You should not penalise learners for making simple numerical errors. You can set an appropriate threshold score for assessing the unit.

You should provide learners with a formulae sheet appropriate to the content of the unit for them to use during the assessment. Learners must not use computer algebra in the assessment.

It is the learners' responsibility to ensure that any calculators they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, calculators must not have retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Using a range of assessment methods helps learners to develop different skills that should be transferable to work or further and higher education.

Opportunities for e-assessment

Assessment that is supported by information and communication technology (ICT), such as e-testing or the use of e-portfolios or social software, may be appropriate for some assessments in this unit.

If you want to use e-assessment, you must ensure that you apply the national standard to all evidence and that conditions of assessment (as specified in the evidence requirements) are met, regardless of the mode of gathering evidence.

Equality and inclusion

This unit is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

You should take into account the needs of individual learners when planning learning experiences, selecting assessment methods or considering alternative evidence.

Guidance on assessment arrangements for disabled learners and/or those with additional support needs is available on the assessment arrangements web page.

Information for learners

Engineering Mathematics 4 (SCQF level 8)

This information explains:

- what the unit is about
- what you should know or be able to do before you start
- what you need to do during the unit
- opportunities for further learning and employment

Unit information

This unit is one of five mathematics units developed for the Higher National Certificate (HNC) and Higher National Diploma (HND) in Engineering. The units help you develop the mathematical skills you need for workplace roles and for more advanced studies in engineering, for example to progress to degree study.

You learn about complex numbers (otherwise known as imaginary numbers) and the algebra that underpins their use. Complex numbers arise in many branches of engineering (for example, AC electrical circuits). You also learn how to apply matrix techniques, which are particularly useful to solve systems of linear equations containing several variables. You develop the knowledge, understanding and skills to use Taylor's and Maclaurin's series to represent functions, and finally you are introduced to a range of techniques for solving an important class of equations known as first-order differential equations, which arise extensively in many areas of engineering.

The unit is likely to involve significant teaching input from your lecturer. This is supplemented by tutorial exercises that allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes you cover to a range of engineering problems. If appropriate, they could choose to teach the unit as a series of lectures supported by tutorial sessions to help you prepare for degree-level study.

You are assessed by an examination under closed-book, controlled and invigilated conditions.

To take the unit, you are expected to have passed the SCQF level 7 unit Engineering Mathematics 3 or an equivalent qualification.

Meta-skills

Throughout the unit, you can develop meta-skills to enhance your employability in the engineering sector.

These skills include self-management, social intelligence and innovation.

Self-management

You develop the meta-skills of focusing, adapting and initiative as you solve engineering problems.

Social intelligence

You develop the meta-skill of communicating as you ask questions and receive information from lecturers.

Innovation

You develop the meta-skills of curiosity, sense-making and critical thinking as you apply mathematical techniques to problem solving.

Administrative information

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Superclass: RB

History of changes

Version	Description of change	Date

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.