Next Generation Higher National Unit Specification

Engineering Mathematics 3 (SCQF level 7)

Unit code: J7L9 47

SCQF level: 7 (8 SCQF credit points)

Valid from: session 2024 to 25

Prototype unit specification for use in pilot delivery only (version 1.0) February 2024

This unit specification provides detailed information about the unit to ensure consistent and transparent assessment year on year.

This unit specification is for teachers and lecturers and contains all the mandatory information required to deliver and assess the unit.

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This edition: February 2024 (version 1.0)

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Unit purpose

This unit develops the breadth of learners' mathematical skills so they can use the Higher National Diploma (HND) in Engineering as a pathway to further studies in Mathematics, including degree study. The unit provides learners with opportunities to develop the knowledge, understanding and skills to apply a range of differential and integral calculus techniques to solve mathematical problems.

The target learner group for the unit is learners who want to further develop their knowledge of mathematics to support a career in engineering fields such as:

- electrical engineering
- mechanical engineering
- systems engineering
- manufacturing engineering
- measurement and control engineering

It is also aimed at learners who want to develop the practical, personal and professional skills required for a successful career as an engineering technician.

Entry to the unit is at your centre's discretion. However, we recommend that learners have knowledge and understanding of basic differentiation and integration techniques, together with sound algebraic skills. This may be evidenced by having completed the SCQF level 7 unit Engineering Mathematics 2 or Higher Mathematics.

Unit outcomes

Learners who complete this unit can:

1 use differentiation techniques to solve mathematical problems
2 use integration techniques to solve mathematical problems

Evidence requirements

You should use a sampling approach to assess knowledge and skills. Learners must provide written or recorded oral evidence to demonstrate their knowledge and skills across all outcomes. Where evidence for outcomes is assessed on a sample basis, you must teach the whole of the content listed in the 'Knowledge and skills' section and make it available for assessment.

We recommend you assess both outcomes using a single end-of-unit assessment. You must use different assessment instruments, and a different sample of knowledge and skills, for re-assessments.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions. Learners must not use computer algebra in assessments for the unit.

To successfully achieve the unit, learners must provide written and/or oral recorded evidence that they have completed the following outcomes.

Outcome 1

Sample any three of the five required items:

- Solve one problem that requires the use of the product rule and one problem that requires the use of the quotient rule.
- Solve one problem that involves implicit differentiation.
- Solve one problem that involves parametric differentiation (where t either can be eliminated or cannot be eliminated).
- Solve one problem that involves the differentiation of an inverse trigonometric function.
- Solve one optimisation problem using at least one of the techniques in the bullet points above.

Outcome 2

Sample any three of the five required items:

- Represent in partial fraction form and integrate any two of the following:
  - a proper fraction with linear factors
  - a proper fraction with recurring linear factors
  - a proper fraction containing a quadratic factor
  - improper fractions
- Solve one definite integral that has an infinite limit.
- Solve one indefinite integral or one definite integral by the method of substitution.
- Solve one problem involving the integration of the product of two functions using integration by parts (the problem can involve either an indefinite or a definite integral).
- Use integration techniques to solve one problem that involves finding the volume of an object or the length of a curve.

Knowledge and skills

The following table shows the knowledge and skills covered by the unit outcomes:

Knowledge	Skills
Outcome 1 Learners should understand: ♦ product and quotient rules ♦ implicit differentiation ♦ parametric differentiation ♦ differentiation of inverse trigonometric functions ♦ optimisation	**Outcome 1** Learners can: ♦ solve one problem that requires the use of the product rule and one problem that requires the use of the quotient rule ♦ solve one problem that involves implicit differentiation ♦ solve one problem that involves parametric differentiation (where t either can be eliminated or cannot be eliminated) ♦ solve one problem that involves the differentiation of an inverse trigonometric function ♦ solve one optimisation problem using at least one of the techniques in the bullet points above

Knowledge	Skills
Outcome 2 Learners should understand: ♦ partial fractions ♦ integrals with infinite limits ♦ integration by substitution ♦ integration by parts ♦ volumes of revolution and curved lengths	**Outcome 2** Learners can: ♦ represent in partial fraction form and integrate any two of the following: — a proper fraction with linear factors — a proper fraction with recurring linear factors — a proper fraction containing a quadratic factor — improper fractions ♦ solve one definite integral that has an infinite limit ♦ solve one indefinite integral or one definite integral by the method of substitution ♦ solve one problem involving the integration of the product of two functions using integration by parts (the problem may involve either an indefinite or definite integral) ♦ use integration techniques to solve one problem that involves finding the volume of an object or the length of a curve

Meta-skills

Throughout the unit, learners develop meta-skills to enhance their employability in the engineering sector.

Self-management

Learners develop the meta-skills of focusing, adapting and initiative as they solve engineering problems.

Social intelligence

Learners develop the meta-skill of communicating as they ask questions and receive information from lecturers.

Innovation

Learners develop the meta-skills of curiosity, sense-making and critical thinking as they apply mathematical techniques to problem solving.

Literacies

Learners develop core skills in the following literacies:

Numeracy

Learners develop their numeracy skills by solving engineering problems using mathematics.

Communication

Learners develop their communication skills by studying the course material, and engaging with other learners, and their teachers or lecturers.

Digital

Learners develop digital literacy by accessing the course material through a virtual learning environment (VLE) and using software tools to solve engineering problems.

Delivery of unit

This unit provides the core mathematical principles and processes that underpin Higher National Certificate (HNC) and HND Engineering units across a range of disciplines. We recommend you deliver the unit towards the beginning of these awards.

The unit is one of a suite of five units in mathematics developed for Higher National Qualifications across a range of engineering disciplines. The five units are:

- Engineering Mathematics 1 at SCQF level 6
- Engineering Mathematics 2 at SCQF level 7
- Engineering Mathematics 3 at SCQF level 7
- Engineering Mathematics 4 at SCQF level 8
- Engineering Mathematics 5 at SCQF level 8

While the exact time allocated to this unit is at your centre's discretion, the notional design length is 40 hours. We suggest the following distribution of time, including assessment:

Outcome 1 — Use differentiation techniques to solve mathematical problems (13 hours)

Outcome 2 — Use integration techniques to solve mathematical problems (10 hours)

Additional guidance

The guidance in this section is not mandatory.

Content and context for this unit

We strongly recommend you use the following list of topics to ensure continuity of learning and teaching.

Use differentiation techniques to solve mathematical problems (outcome 1)

- State the product and quotient rules. For example:

For $y(x) = u(x)v(x)$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} \quad \text{or} \quad y' = vu' + uv'$$

and for $y(x) = \frac{u(x)}{v(x)}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \text{ or } \frac{vu' - uv'}{v^2}$$

- Solve problems involving the use of the product and quotient rules. For example:

$x^2 \sin x$, $(4x+9)e^{-3x}$, $\frac{t^2-1}{t^2+1}$, $\frac{e^{2x}\cos x}{x^3}$

- Explain the difference between y being expressed explicitly in terms of x, and y being expressed implicitly in terms of x.
- Solve a range of problems involving implicit differentiation. For example:
  $x^2 + 3y^2 + 4x - 5y = 7$
- Extend to higher order differentials if time allows.
- Explain what is meant by a parameter and parametric differentiation.
- Solve parametric differentiation problems where t can be eliminated and where it cannot be eliminated. For example: $x = 1 - t$, $y = 2t^2 + 5t + 7$, $y = t^3 + \cos t$, $e^t + t$
- Identify the derivatives for inverse trigonometric functions on a table of standard derivatives.
- Solve a range of problems involving the differentiation of functions that include inverse trigonometric functions.
- Apply differentiation to optimise a parameter or parameters of a problem using at least one of the differentiation techniques used in the outcome.

Use integration techniques to solve mathematical problems (outcome 2)

- Explain that partial fractions involve breaking down complicated fractions into the sum of simpler fractions.
- Explain the difference between proper and improper fractions.
- Solve a range of integration problems that involve the partial fraction representations of the following forms of fractions:
  - a proper fraction with linear factors
  - a proper fraction with recurring linear factors
  - a proper fraction containing a quadratic factor
  - improper fractions
- Solve integrals with infinite limits of integration. For example:

$$\int_0^{\infty} e^{-x}\,dx$$

- Solve a range of indefinite and definite integrals using the method of substitution. For example:

$$\int (5x+2)^6\,dx \ , \ \int \cos(4x-1)\,dx \ , \ \int x\sqrt{2x+1dx} \ , \ \int \frac{x}{\sqrt{1-x^2}}\,dx$$

- Solve problems involving the integration of the product of two functions using the following formula (integration by parts):

$$\int u\left(\frac{dv}{dx}\right)dx \ = \ uv - \int v\left(\frac{du}{dx}\right)dx$$

For example: $\int x^2 e^{2x}\,dx$, $\int e^x \sin x\,dx$, $\int 3\ln x\,dx$

- Solve integration problems involving volumes of revolution, such as volumes of cones or spheres.
- Solve problems involving the length of curves using the following formula:

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Approaches to delivery

The unit provides many of the core mathematical principles and processes learners need to study engineering at a more advanced level. Given the subject matter, we recommend you deliver this unit after learners have studied Engineering Mathematics 2.

You can deliver the outcomes in any order. However, we recommend you deliver outcome 1 first, followed by outcome 2.

We recommend that you deliver the unit using a mainly didactic approach. You should supplement teaching with formative assessment where learners have opportunities to develop their knowledge, understanding and skills of the differentiation and integration techniques covered in the unit. You can use computer software and computer algebra to support learning (for example, to confirm the solutions of mathematical problems), but we strongly recommend that these learning resources are only used in a supportive capacity and not as the principal means of delivering unit content.

Approaches to assessment

You can use different types of assessment to generate evidence.

We recommend using an examination question paper. This should comprise an appropriate balance of short-answer, restricted-response and structured questions.

Learners should not see the assessment papers before the assessment takes place. You must ensure the security, integrity and confidentiality of the assessment papers at all times. You should conduct assessments under closed-book, controlled and invigilated conditions.

You should not group or label the examination questions by outcome if you use a single end-of-unit examination.

The total assessment time for both outcomes — whether assessed individually or during a single assessment — should not exceed two hours. When you assess learners' responses in a summative assessment, you should principally concentrate on their ability to apply the correct mathematical techniques and processes. You should not penalise learners for making simple numerical errors. You can set an appropriate threshold score for assessing the unit. You should use a threshold score for each assessment if you are using outcome-level assessment.

You should provide learners with a formulae sheet appropriate to the content of the unit for them to use during the assessment. Learners must not use computer algebra in the assessment.

It is the learners' responsibility to ensure that any calculators they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, calculators must not have retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Using a range of assessment methods helps learners to develop different skills that should be transferable to work or further and higher education.

Opportunities for e-assessment

Assessment that is supported by information and communication technology (ICT), such as e-testing or the use of e-portfolios or social software, may be appropriate for some assessments in this unit.

If you want to use e-assessment, you must ensure that you apply the national standard to all evidence and that conditions of assessment (as specified in the evidence requirements) are met, regardless of the mode of gathering evidence.

Equality and inclusion

This unit is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

You should take into account the needs of individual learners when planning learning experiences, selecting assessment methods or considering alternative evidence.

Guidance on assessment arrangements for disabled learners and/or those with additional support needs is available on the assessment arrangements web page.

Information for learners

Engineering Mathematics 3 (SCQF level 7)

This information explains:

- what the unit is about
- what you should know or be able to do before you start
- what you need to do during the unit
- opportunities for further learning and employment

Unit information

This unit is one of five mathematics units developed for the Higher National Certificate (HNC) and Higher National Diploma (HND) in Engineering. These units help you develop the mathematical skills you need for workplace roles and more advanced studies in engineering, for example to progress to degree study.

You learn to use a range of differential calculus techniques such as the product and quotient rules, implicit and parametric differentiation, and differentiation involving inverse trigonometrical functions. You use these differentiation techniques to find the optimum solution to problems. In addition, you develop the knowledge and skills to break down complicated polynomial fractions into partial fractions that allow these complex fractions to be integrated in a more straightforward way. You also learn to use the method of substitution and integration by parts integration techniques. You use the integration techniques you have learnt in the unit to solve volume of revolution and length of a curve problems.

It is likely that the unit involves significant teaching input from your lecturer. This is supplemented by tutorial exercises that allow you to develop the knowledge, understanding and skills to apply the mathematical principles and processes you cover to a range of engineering problems.

You could be assessed on an outcome-by-outcome basis or by a single assessment. All assessments are under closed-book, controlled and invigilated conditions.

To take the unit, you are expected to have passed the SCQF level 7 unit Engineering Mathematics 2 or an equivalent qualification.

Meta-skills

Throughout the unit, you can develop meta-skills to enhance your employability in the engineering sector.

These skills include self-management, social intelligence and innovation.

Self-management

You develop the meta-skills of focusing, adapting and initiative as you solve engineering problems.

Social intelligence

You develop the meta-skill of communicating as you ask questions and receive information from your lecturers.

Innovation

You develop the meta-skills of curiosity, sense-making and critical thinking as you apply mathematical techniques to problem solving.

Administrative information

Published: February 2024 (version 1.0)

Superclass: RB

History of changes

Version	Description of change	Date

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.

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