# Next Generation Higher National Unit Specification

## Engineering Mathematics 2 (SCQF level 7)

**Unit code:** J7GL 47

**SCQF level:** 7 (8 SCQF credit points)

**Valid from:** session 2023–24

## Prototype unit specification for use in pilot delivery only (version 1.0) August 2023

This unit specification provides detailed information about the unit to ensure consistent and transparent assessment year on year.

This unit specification is for teachers and lecturers and contains all the mandatory information required to deliver and assess the unit.

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This edition: August 2023 (version 1.0)

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# Unit purpose

This unit further develops the mathematical skills learners developed in Engineering Mathematics 1 at SCQF level 6. This unit provides opportunities for learners to develop knowledge, understanding and skills to solve problems involving trigonometric and hyperbolic functions and identities, differentiate and integrate a wide range of functions, and use differentiation and integration techniques to solve engineering problems.

The target learner group for the unit is learners who want to further develop their knowledge of mathematics to support a career in engineering fields such as:

- electrical engineering
- mechanical engineering
- systems engineering
- manufacturing engineering
- measurement and control engineering

It is also aimed at learners who want to develop the practical, personal and professional skills required for a successful career as an engineering technician. At SCQF level 7, the unit provides learners with suitable knowledge and skills for progression or articulation to further study.

Entry to the unit is at your centre's discretion. However, we strongly recommend that learners have completed the unit Engineering Mathematics 1 at SCQF level 6.

# Unit outcomes

Learners who complete this unit can:

1 solve trigonometric and hyperbolic function problems
2 use differentiation techniques to solve engineering problems
3 use integration techniques to solve engineering problems

## Evidence requirements

You should use a sampling approach to assess knowledge and skills. Learners must provide written or recorded oral evidence to demonstrate their knowledge and skills across all outcomes by showing that they can do the following.

### Outcome 1

Learners must provide evidence of three out of the five knowledge and skills in this outcome. They should provide the following evidence for the knowledge and skills items sampled:

- Evaluate any two of the following trigonometric functions: $\sec\alpha$, $\operatorname{cosec}\alpha$, or $\cot\alpha$ for a given value ($\S$) of $\alpha$
- Solve one problem using one of the following compound angle formulae: $\sin(x \pm \beta)$ or $\cos(x \pm \beta)$
- Solve one problem using one or more of the following trigonometric identities:

$$\sin^2\alpha + \cos^2\alpha = 1$$
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = 2\cos^2\alpha - 1$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

- Evaluate any two of the following hyperbolic functions: $\sinh\alpha$, $\cosh\alpha$ or $\tanh\alpha$ for a given value ($_s$) of $_\alpha$
- Solve one problem involving hyperbolic identities.

### Outcome 2

Learners must provide evidence of three out of the five knowledge and skills in this outcome. They should provide the following evidence for the knowledge and skills items sampled:

- Use standard derivatives to solve two problems involving differentiation (standard derivatives to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$).
- Differentiate a function that requires the use of the chain rule.
- Apply first and second derivatives to determine the position and nature of a turning point on a curve.
- Use differentiation to determine the rate of change of a variable in an engineering problem.

- Apply differentiation techniques to find the optimum solution to a problem.

## Outcome 3

Learners must provide evidence of two out of the three knowledge and skills in this outcome. They should provide the following evidence for the knowledge and skills items sampled:

- Solve one indefinite and one definite integral.
- Solve two integrals using integrals of standard functions (standard functions to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$).
- Apply integration techniques to the solution of an engineering problem.

We recommend that you assess all three outcomes in a single end-of-unit assessment. You can also assess outcomes individually. You must use different assessment instruments, and a different sample of knowledge and skills for re-assessments.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions.

Learners must not use computer algebra in the assessment.

# Knowledge and skills

The following table shows the knowledge and skills covered by the unit outcomes:

| Knowledge | Skills |
|---|---|
| **Outcome 1**<br>Learners should understand:<br><br>♦ how to solve trigonometric and hyperbolic function problems | **Outcome 1**<br>Learners can:<br><br>♦ evaluate any two of the following trigonometric functions: $\sec\alpha$, $\operatorname{cosec}\alpha$, or $\cot\alpha$ for a given value (ʃ) of $\alpha$<br>♦ solve one problem using one of the following compound angle formulae: $\sin(x \pm \beta)$ or $\cos(x \pm \beta)$<br>♦ solve one problem using one or more of the following trigonometric identities:<br><br>$\sin^2\alpha + \cos^2\alpha = 1$<br>$\sin 2\alpha = 2\sin\alpha\cos\alpha$<br>$\cos 2\alpha = 2\cos^2\alpha - 1$<br>$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$<br><br>♦ evaluate any two of the following hyperbolic functions: $\sinh\alpha$, $\cosh\alpha$ or $\tanh\alpha$ for a given value (ʃ) of $\alpha$<br>♦ solve one problem involving hyperbolic identities |

| Knowledge | Skills |
|---|---|
| **Outcome 2**<br>Learners should understand:<br><br>♦ how to use differentiation techniques to solve engineering problems | **Outcome 2**<br>Learners can:<br><br>♦ use standard derivatives to solve two problems involving differentiation (standard derivatives to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$)<br>♦ differentiate a function that requires the use of the chain rule<br>♦ apply first and second derivatives to determine the position and nature of a turning point on a curve<br>♦ use differentiation to determine the rate of change of a variable in an engineering problem<br>♦ apply differentiation techniques to find the optimum solution to a problem |
| **Outcome 3**<br>Learners should understand:<br><br>♦ how to use integration techniques to solve engineering problems | **Outcome 3**<br>Learners can:<br><br>♦ solve one indefinite and one definite integral<br>♦ solve two integrals using integrals of standard functions (standard functions to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$)<br>♦ apply integration techniques to the solution of an engineering problem |

# Meta-skills

Throughout the unit, learners develop meta-skills to enhance their employability in the engineering sector.

## Self-management

Learners develop the meta-skills of focusing, adapting and initiative as they solve engineering problems.

## Social intelligence

Learners develop the meta-skill of communicating when enquiring about or submitting evidence requirements for each outcome.

## Innovation

Learners develop the meta-skills of curiosity, sense-making and critical thinking as they apply mathematical techniques to problem solving.

# Literacies

Learners develop core skills in the following literacies:

## Numeracy

Learners develop numeracy skills by performing engineering calculations.

## Communication

Learners develop their communication skills by reporting and presenting results for all outcomes.

## Digital

Learners develop their digital literacy by using research methods. They use software for engineering mathematics applications.

# Delivery of unit

This unit provides the core mathematical principles and processes that underpin Higher National Certificate (HNC) and Higher National Diploma (HND) Engineering units across a range of disciplines.

The unit is one of a suite of five units in mathematics developed for Higher National Qualifications across a range of engineering disciplines. The five units are:

- Engineering Mathematics 1 at SCQF level 6
- Engineering Mathematics 2 at SCQF level 7
- Engineering Mathematics 3 at SCQF level 7
- Engineering Mathematics 4 at SCQF level 8
- Engineering Mathematics 5 at SCQF level 8

The amount of time you allocate to this unit is at your centre's discretion. However, the notional design length is 40 hours.

The amount of time you allocate to each outcome is also at your discretion. We suggest the following distribution of time, including assessment:

**Outcome 1** — Solve trigonometric and hyperbolic function problems (12 hours)

**Outcome 2** — Use differentiation techniques to solve engineering problems (10 hours)

**Outcome 3** — Use integration techniques to solve engineering problems (8 hours)

# Additional guidance

The guidance in this section is not mandatory.

## Content and context for this unit

We strongly recommend you use the following list of topics to ensure continuity of learning and teaching, and that learners are prepared for assessment.

### Solve trigonometric and hyperbolic function problems (outcome 1)

- Define secant, cosecant and cotangent ratios.
- Evaluate secant, cosecant and cotangent ratios for given angles.
- Distinguish between secant, cosecant and cotangent and $\cos^{-1}$, $\sin^{-1}$ and $\tan^{-1}$
- State compound angle formulae (for example, $\sin(\alpha+\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha$ and $\cos(\alpha\ \pm\ \beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$
- Apply compound angle formulae to trigonometrical problems (for example, $\sin(180^\circ+\theta)=-\sin\theta$
- State $\sin^2\alpha+\cos^2\alpha=1$, $\sin 2\alpha=2\sin\alpha\cos\alpha$ and $\cos 2\alpha=2\cos^2\alpha-1$ or $1-2\sin^2\alpha$, $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$
- Use the trigonometrical equations in the previous bullet point to simplify trigonometrical identities and solve trigonometrical equations.
- Define $\sinh x$, $\cosh x$, $\tanh x$, $\operatorname{cosech} x$, $\operatorname{sech} x$ and $\coth x$
- Use the following hyperbolic identities to prove identities and modify equations containing $e^x$ and $e^{-x}$:

$$e^x=\cosh x+\sinh x$$
$$e^{-x}=\cosh x-\sinh x$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$$
$$\cosh(x\ \pm\ y)=\cosh x\cosh\ y\pm\sinh x\sinh y$$
$$\sinh 2x=2\sinh x\cosh x$$
$$\cosh 2x=\cosh^2 x+\sinh^2 x$$

### Use differentiation techniques to solve engineering problems (outcome 2)

- Revise indices including negative and fractional indices.
- Introduce the concept of differentiation from first principles (not assessable).
- Introduce standard derivatives to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$
- Use standard derivatives to find the derivatives of functions containing one or more of the terms in the previous bullet point.

- State the chain rule, for example:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

- Apply the chain rule to functions such as $\left(3x^4 + 7\right)^3$, $\sin\left(t^2 + 1\right)$, $5e^{\sin u}$
- Define higher derivatives (second, third, etc.).
- Use the first and second derivatives to find the maximum and minimum of a function.
- Use differentiation to evaluate rates of change problems in engineering.
- Apply differentiation to optimise a parameter or parameters of a problem (for example, the condition under which the maximum electrical power will be transferred from a voltage source to load).

### Use integration techniques to solve engineering problems (outcome 3)

- Define what is meant by integration (for example, as anti-differentiation, as the area bounded by curves).
- Define indefinite and definite integrals.
- Solve indefinite and definite integrals using standard integrals (standard integrals to include $ax^n$, $(ax+b)^n$, trigonometric, hyperbolic, $\ln(ax+b)$ and $e^{(ax+b)}$).
- Apply integration to solve problems in engineering (area under a velocity–time curve giving distance travelled, work done by an expanding gas, first and second moments of area, centroids, mean values, root mean square values, etc.).

## Approaches to delivery

You can deliver the outcomes in any order. However, we recommend that you deliver outcome 1 first, followed by outcome 2 and then outcome 3.

We recommend that you deliver this unit using a mainly didactic approach. You should supplement teaching with formative assessment where learners have opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the unit. You can use computer software and computer algebra to support learning (for example, to confirm the solutions of mathematical problems), but we strongly recommend that these learning resources are only used in a supportive capacity and not as the principal means of delivering unit content.

If you deliver the unit as part of a group award, you should teach and assess it within that group award's subject area.

Acceptable performance in the unit is the satisfactory achievement of the standards set out in this part of the unit specification. All sections of the statement of standards are mandatory and cannot be altered without reference to SQA.

## Approaches to assessment

You can use different types of assessment to generate evidence.

We recommend using an examination question paper. This should comprise an appropriate balance of short-answer, restricted-response and structured questions.

Learners should not see the assessment papers before the assessment takes place. You must ensure the security, integrity and confidentiality of the assessment papers at all times. You should conduct assessments under closed-book, controlled and invigilated conditions.

Where evidence for outcomes is assessed on a sample basis, you must teach the whole of the content listed in the 'Knowledge and skills' section and make it available for assessment. Learners should not know in advance the items that will be in the assessment, and you should use different items for each assessment occasion.

You should not group or label the examination questions by outcome if you use a single end-of-unit examination.

The total assessment time for all three outcomes — whether assessed individually or during a single assessment — should not exceed two hours. When you assess learners' responses in a summative assessment, you should principally concentrate on their ability to apply the correct mathematical techniques and processes. You should not penalise learners for making simple numerical errors. You can set an appropriate threshold score for assessing the unit. You should use a threshold score for each assessment if you are using outcome-level assessment.

You should provide learners with a formulae sheet appropriate to the content of the unit for them to use during the assessment. Learners must not use computer algebra in the assessment.

Learners must ensure that any calculators they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- symbolic differentiation or integration
- communication with other machines or the internet

In addition, calculators must not have retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematic formulae

Prior verification of centre-devised assessments helps ensure that the national standard is being met. Using a range of assessment methods helps learners to develop different skills that should be transferable to work or further and higher education.

# Equality and inclusion

This unit is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

You should take into account the needs of individual learners when planning learning experiences, selecting assessment methods or considering alternative evidence.

Guidance on assessment arrangements for disabled learners and/or those with additional support needs is available on the assessment arrangements web page: www.sqa.org.uk/assessmentarrangements.

# Information for learners

## Engineering Mathematics 2 (SCQF level 7)

This information explains:

- what the unit is about
- what you should know or be able to do before you start
- what you need to do during the unit
- opportunities for further learning and employment

## Unit information

This unit is one of five mathematics units developed for the Higher National Certificate (HNC) and Higher National Diploma (HND) in Engineering. These units help you develop the mathematical skills you need for workplace roles and more advanced studies in engineering, for example to progress to degree study at university.

The unit introduces you to differential calculus, which is widely used to solve engineering problems. This includes differentiating functions using standard derivatives, determining rates of change and finding optimum solutions to engineering problems. You are also introduced to integral calculus. You learn to solve indefinite and definite integrals using standard integrals, and use integration to solve engineering problems.

It is likely that the unit involves significant teaching input from your lecturer. This is supplemented by tutorial exercises that allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes you cover to a range of engineering problems.

You could be assessed on an outcome-by-outcome basis or by a single assessment. All assessments are under closed-book, controlled and invigilated conditions.

To take the unit, you are expected to have passed the SCQF level 6 unit Engineering Mathematics 1 or an equivalent qualification.

# Administrative information

**Published:** August 2023 (version 1.0)

**Superclass:** RB

## History of changes

| Version | Description of change | Date |
|---|---|---|
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