Next Generation Higher National Unit Specification

Engineering Mathematics 1 (SCQF level 6)

Unit code: J7GK 46

SCQF level: 6 (8 SCQF credit points)

Valid from: session 2023–24

Prototype unit specification for use in pilot delivery only (version 1.0) August 2023

This unit specification provides detailed information about the unit to ensure consistent and transparent assessment year on year.

This unit specification is for teachers and lecturers and contains all the mandatory information required to deliver and assess the unit.

This edition: August 2023 (version 1.0)

Unit purpose

This unit develops and consolidates the basic level of mathematical skills learners need for a range of engineering disciplines. It provides learners with opportunities to develop knowledge, understanding and skills to solve mathematical problems, including polynomial, trigonometrical, exponential and logarithmic functions. It also introduces learners to 3-dimensional vectors and complex numbers.

The target learner group for the unit is learners who want to develop their knowledge of mathematics to support a career in engineering fields such as:

- electrical engineering
- mechanical engineering
- systems engineering
- manufacturing engineering
- measurement and control engineering

It is also aimed at learners who want to develop the practical, personal and professional skills required for a successful career as an engineering technician.

Entry to the unit is at your centre's discretion. However, we recommend that learners already have good algebraic skills, and can transpose engineering formulae and solve simple trigonometric equations. For example, they may have completed the National Unit Mathematics: Technician 1 at SCQF level 6, or National 5 Mathematics.

Unit outcomes

Learners who complete this unit can:

1 solve problems involving functions and trigonometric equations
2 solve problems involving exponential and logarithmic equations
3 apply mathematical techniques involving vectors and complex numbers

Evidence requirements

Use a sampling approach to assess knowledge and skills.

To successfully achieve the unit, learners must provide written and/or oral evidence for the following outcomes.

Outcome 1

Provide evidence of the following knowledge and/or skills each time this outcome is assessed:

- Solve two problems (one in degrees and one in radians) involving expressions of the form $A\sin(nx \pm a) = b$ or $A\cos(nx \pm a) = b$

Provide evidence of two out of three knowledge and/or skills remaining in this outcome. The following evidence should be provided for the particular knowledge and/or skill items sampled:

- Identify the domain, range and asymptotes of a given function.
- Solve one problem involving an inverse function.
- Solve one problem involving a composite function.

Outcome 2

Provide evidence of the following knowledge and/or skills each time this outcome is assessed.

Solve three problems to cover the knowledge and/or skills as follows, combining them where appropriate:

- Evaluate formulae; one must include exponentials and one must include logarithms.
- Transpose expressions from exponential form to logarithmic form and vice versa.
- Use two out of the three laws of logarithms:
  — $\log x + \log y = \log xy$
  — $\log x - \log y = \log \frac{x}{y}$
  — $p \log x = \log x^p$
- Sketch or identify a graph of either a logarithmic function (of the form $y = A\ In\ x$ or $y = A\log_{10} x$) or an exponential function (of the form Ae^x or $A.10^x$).

Fractional and negative coefficients and indices should be included.

Outcome 3

Provide evidence of five out of seven knowledge and/or skills in this outcome. The following evidence should be provided for the particular knowledge and/or skills items sampled:

- Verify that three 3-dimensional coordinates (A,B,C) are collinear and determine the ratio of AB to BC.
- Solve two vector problems involving a combination of at least two of the following: addition of vectors; subtraction of vectors; and multiplication of a vector by a scalar.
- Calculate the angle between two 3-dimensional vectors using the scalar product.
- Convert one vector (2-dimensional cases) or complex number in Cartesian form to polar form, and one vector or complex number in polar form to Cartesian form, clearly showing all working.
- Solve one problem involving the addition and subtraction of complex numbers (in rectangular form).
- Solve one problem involving the multiplication of two complex numbers (in rectangular form) and one problem involving the division of two complex numbers (in polar form).
- Represent accurately complex numbers in all four quadrants of an Argand diagram.

We recommend that you assess all three outcomes in a single end-of-unit assessment. You can also assess outcomes individually. You must use different assessment instruments, and a different sample of knowledge and skills for re-assessments.

All assessments should be unseen, closed-book and carried out under supervised, controlled conditions.

Learners must not use computer algebra in the assessment.

Knowledge and skills

The following table shows the knowledge and skills covered by the unit outcomes:

Knowledge	Skills
Outcome 1 Learners should understand: ♦ functions and trigonometric equations	**Outcome 1** Learners can: ♦ solve two problems (one in degrees and one in radians) involving expressions of the form $A\sin(nx \pm a) = b$ or $A\cos(nx \pm a) = b$ ♦ provide evidence for two of the following: — identify the domain, range and asymptotes of a given function — solve one problem involving an inverse function — solve one problem involving a composite function
Outcome 2 Learners should understand: ♦ exponential and logarithmic equations	**Outcome 2** Learners can: ♦ solve problems involving exponential and logarithmic equations of the following: — evaluation of logarithmic and exponential expressions — transposition from logarithmic to exponential form and vice versa — laws of logarithms — graphics of logarithmic and exponential functions

Knowledge	Skills
Outcome 3 Learners should understand: ♦ vectors and complex numbers	**Outcome 3** Learners can: ♦ apply mathematical techniques involving vectors and complex numbers of the following: — collinearity — addition, subtraction and scalar multiplication of vectors — scalar product — conversion of complex numbers between rectangular and polar form — addition and subtraction of complex numbers — multiplication and division of complex numbers — representation of complex numbers on an Argand diagram

Meta-skills

Throughout the unit, learners develop meta-skills to enhance their employability in the engineering sector.

Self-management

Learners develop the meta-skills of focusing, adapting and initiative as they solve engineering problems.

Social intelligence

Learners develop the meta-skill of communicating when enquiring about or submitting evidence requirements for each outcome.

Innovation

Learners develop the meta-skills of curiosity, sense-making and critical thinking as they apply mathematical techniques to problem solving.

Literacies

Learners develop core skills in the following literacies:

Numeracy

Learners develop numeracy skills by performing engineering calculations.

Communication

Learners develop their communication skills by reporting and presenting results for all outcomes.

Digital

Learners develop their digital literacy by using research methods. They use software for engineering mathematics applications.

Delivery of unit

This unit provides core mathematical principles and processes that underpin Higher National Certificate (HNC) and Higher National Diploma (HND) Engineering units across a range of disciplines. We recommend you deliver the unit towards the beginning of these awards.

This unit is one of a suite of five units in mathematics developed for Higher National Qualifications across a range of engineering disciplines. The five units are:

- Engineering Mathematics 1 at SCQF level 6
- Engineering Mathematics 2 at SCQF level 7
- Engineering Mathematics 3 at SCQF level 7
- Engineering Mathematics 4 at SCQF level 8
- Engineering Mathematics 5 at SCQF level 8

The amount of time you allocate to this unit is at your centre's discretion. However, the notional design length is 40 hours.

The amount of time you allocate to each outcome is also at your discretion. We suggest the following distribution of time, including assessment:

Outcome 1 — Solve problems involving functions and trigonometric equations (12 hours)

Outcome 2 — Solve problems involving exponential and logarithmic equations (12 hours)

Outcome 3 — Apply mathematical techniques involving vectors and complex numbers (16 hours)

Additional guidance

The guidance in this section is not mandatory.

Content and context for this unit

We strongly recommend you use the following list of topics to ensure continuity of learning and teaching, and that learners are prepared for assessment.

Solve problems involving functions and trigonometric equations (outcome 1)

- Introduce the concept of the function. You may do this in terms of set theory and, in particular, a function or mapping as a rule that relates each element in one set to one and only one element in another set.
- List typical functions that occur in engineering.
- Introduce typical mathematical notation used with functions (for example, $f(x)$, $x \varepsilon R$, (or N, W, Z, Q etc).
- Explain independent and dependent variables.
- Define the terms 'domain' and 'range of a function'.
- Introduce the concept of a pole (s) in a function and explain the impact of a pole (s) on the domain of a function that has one or more poles.
- Demonstrate the way to determine the range and domains of functions.
- Define the term 'asymptotes'.
- Demonstrate the way to determine the asymptotes to functions.
- Introduce the idea of an inverse function $f^{1}(x)$
- Determine the inverse functions of a range of functions.
- Graph functions and their inverse functions.
- Introduce composite functions. Again, you may do this in terms of set theory.
- Derive and evaluate a number of composite functions.
- Introduce the concept of the radian.
- Convert degrees to radians and vice versa (for example, $\text{rad} = \frac{\theta}{180} \times \pi$)
- Determine the solutions of equations of the form $A\sin(nx+\alpha) = \pm B$ or $A\cos(nx+\alpha) = \pm B$

Transpose and solve exponential and logarithmic equations (outcome 2)

- Introduce the exponential function $y = a^x$
- Plot graphs of this function for different values of a
- Plot a graph of $y = a^{-x}$ to demonstrate that this produces the image of $y = a^x$
- Explain that $x = \log_a y$ is the inverse function of $y = a^x$
- Examine $y = 10^x$ and $x = \log_{10} y$ as an important example of exponential and logarithmic functions.

- Plot $y = 10^x$ and $x = \log_{10} y$ for a specified domain.
- Examine $y = e^x$ and $x = \log_e y$ as a further important example of exponential and logarithmic functions, especially in the context of engineering.
- Plot $y = e^x$ and $x = \log_e y$ for a specified domain.
- Evaluate expressions that include 10^x, $\log_{10} y$, e^x and $\log_{10} y$
- Introduce the three laws of logarithms.
- Solve problems involving the three laws of logarithms.
- Transpose and evaluate expressions from engineering that include the term Ae^{kt} or Ae^{-kt}
- Solve exponential equations $(5^x = 95,\ x^{3\cdot5} = 0\cdot1)$
- Solve logarithmic equations (for example, $\log_{10}(x+2) + \log_{10}(x+4) = 0\cdot5$; $\log_e(t-3) - \log_e(t+3) = \log_e 6$)

Apply mathematical techniques to vectors and complex numbers (outcome 3)

2-dimensional and 3-dimensional vectors

- Introduce the concept of a vector and scalar and give examples of each form from engineering.
- Outline notation commonly used with vectors.
- Demonstrate the way to add and subtract 2-dimensional vectors graphically.
- Introduce 2-dimensional vectors in component form.
- Add and subtract vectors using their component form.
- Multiply 2-dimensional vectors by a scalar.
- Calculate the magnitude of 2-dimensional vectors using the equation:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Introduce 3-dimensionial coordinates and the representation of 3-dimensional vectors in component form.
- Calculate the magnitude of 3-dimensional vectors using the equation:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Apply the concept of collinearity to 2-dimensional and 3-dimensional vectors.
- Introduce the scalar product for 2-dimensional and 3-dimensional vectors.
- Identify the properties of the scalar product (for example, $a.b = b.a$, $a.(b+c) = a.b + a.c$ etc).
- Demonstrate the way in which the scalar product can be used to find the angle between two vectors.

Complex numbers

- Introduce complex numbers. You can take different approaches to this, for example what happens when the determinant, $b^2 - 4ac < 0$, or $i = \sqrt{-1}$ etc.
- Identify normal notation used with complex numbers (engineers tend to use j while mathematicians use i).
- Introduce the Argand diagram.
- Demonstrate the representation of complex numbers in Cartesian and polar form (noting the domain of principal argument, $\arg(z)$).
- Demonstrate the way to convert between Cartesian and polar form and vice versa.
- Add and subtract complex numbers.
- Multiply complex numbers in both Cartesian and polar form.
- Introduce the complex conjugate.
- Divide complex numbers in both Cartesian and polar form.
- Represent complex numbers in all four quadrants of an Argand diagram.

Approaches to delivery

You can deliver the outcomes in any order. However, we recommend that you deliver outcome 1 first, followed by outcome 2 and then outcome 3.

We recommend that you deliver the unit using a mainly didactic approach. You should supplement teaching with formative assessment, where learners have opportunities to develop their knowledge, understanding and skills of the mathematical topics covered in the unit. You can use computer software and computer algebra to support learning (for example, to confirm the solutions of mathematical problems), but we strongly recommend that these learning resources are only used in a supportive capacity and not as the principal means of delivering unit content.

Approaches to assessment

You can use different types of assessment to generate evidence.

We recommend using a question paper. This should comprise an appropriate balance of short-answer, restricted-response and structured questions.

Learners should not see the assessment papers before the assessment takes place. You must ensure the security, integrity and confidentiality of the assessment papers at all times. You should conduct assessments under closed-book, controlled and invigilated conditions.

You should not group or label the assessment questions by outcome if you use a single end-of-unit assessment.

The total assessment time for all three outcomes — whether assessed individually or during a single assessment — should not exceed two hours. When you assess learners' responses in a summative assessment, you should concentrate principally on their ability to apply the correct mathematical technique and processes. You should not penalise learners for making simple numerical errors. You can set an appropriate threshold score for assessing the unit. You should use a threshold score for each assessment if you are using outcome-level assessment.

You should provide learners with a formulae sheet appropriate to the content of the unit for them to use during the assessment. Learners must not use computer algebra in the assessment.

Learners must ensure that any calculators they use during assessment are not designed or adapted to offer any of the following facilities:

- language translators
- symbolic algebra manipulation
- complex number manipulation
- communication with other machines or the internet

In addition, calculators must not have retrievable information stored in them. This includes:

- databanks
- dictionaries
- mathematical formulae

Prior verification of centre-devised assessments helps ensure that the national standard is being met. Using a range of assessment methods helps learners to develop different skills that should be transferable to work or further and higher education.

Equality and inclusion

This unit is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

You should take into account the needs of individual learners when planning learning experiences, selecting assessment methods or considering alternative evidence.

Guidance on assessment arrangements for disabled learners and/or those with additional support needs is available on the assessment arrangements web page: www.sqa.org.uk/assessmentarrangements.

Information for learners

Engineering Mathematics 1 (SCQF level 6)

This information explains:

- what the unit is about
- what you should know or be able to do before you start
- what you need to do during the unit
- opportunities for further learning and employment

Unit information

This unit is one of five mathematics units developed for Higher National Certificate (HNC) and Higher National Diploma (HND) Engineering. These units help you develop the mathematical skills you need for workplace roles and more advanced studies in engineering, for example to progress to degree study at university.

This unit is mandatory in a number of Higher National Certificates in Engineering.

The unit develops and consolidates a basic level of mathematical skills across a range of engineering disciplines. You learn about the properties of some mathematical functions commonly used in engineering including linear, trigonometrical, exponential and logarithmic functions. You also learn how to solve trigonometrical, exponential and logarithmic equations. You study 2-dimensional and 3-dimensional vectors and complex numbers that are widely used in mechanical and electrical engineering.

It is likely that the unit involves significant teaching input from your lecturer. This is supplemented by tutorial exercises that allow you to develop the knowledge, understanding and skills to apply the mathematic principles and processes you cover to a range of engineering problems.

You could be assessed on an outcome-by-outcome basis or by a single assessment. All assessments are under closed-book, controlled and invigilated conditions.

To take the unit, we recommend that you already have good algebraic skills, and can transpose engineering formulae and solve simple trigonometric equations. For example, you may have completed the National Unit Mathematics: Technician 1 at SCQF level 6, or National 5 Mathematics.

Administrative information

Published: August 2023 (version 1.0)

Superclass: RB

History of changes

Version	Description of change	Date

Note: please check SQA's website to ensure you are using the most up-to-date version of this document.