



S874/76/11

Matamataig
Pàipear 1 (Gun Àireamhair)

Deit — Gun bhuinteanas
Ùine — 1 uair 15 mionaid



Comharran gu lèir — 55

Feuch na ceistean UILE.

CHAN FHAOD thu àireamhair a chleachdadh.

Gus na comharran gu lèir fhaighinn, feumaidh tu d' obrachadh a-mach a shealltainn sna freagairtean agad.

Cuir na h-aonadan anns na freagairtean agad far a bheil sin iomchaidh.

Chan fhaigh thu comharran airson fhreagairtean a fhuair thu bho leughadh dhealbhan-sgèile.

Sgrìobh do fhreagairtean gu soilleir anns na beàrnan ann an leabhran nam freagairtean. Chan eil farsaingeachd an àite airson freagairt idir ag innse na bu chòir dhut a sgriobhadh. Cha leig thu a leas an t-àite gu lèir a chleachdadh.

Tha àite a bharrachd ann airson fhreagairtean aig deireadh an leabhrain seo. Ma chleachdas tu an t-àite seo, feumaidh tu àireamh na ceiste a tha thu a' freagairt a chomharrachadh gu soilleir.

Cleachd inc gorm no dubh.

Mus fàg thu seòmar na deuchainne feumaidh tu leabhran nam freagairtean a thoirt don Fhreiceadan; mura dèan thu sin, dh'fhaodadh tu na comharran gu lèir airson a' phàipeir seo a chall.

LIOSTA FHOIRMLEAN

Cearcall

Tha an co-aontar $x^2 + y^2 + 2gx + 2fy + c = 0$ a' riochdachadh meadhan clearcaill $(-g, -f)$ agus radius $\sqrt{g^2 + f^2 - c}$.

Tha an co-aontar $(x-a)^2 + (y-b)^2 = r^2$ a' riochdachadh meadhan clearcaill (a, b) agus radius r .

Toradh scalar

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta, \text{ far a bheil } \theta \text{ a' riochdachadh a' cheàirn eadar } \mathbf{a} \text{ agus } \mathbf{b}$$

no
$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ far a bheil } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ agus } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Foirmlean triantanach

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Clàr dheribheatuibhean àbhaisteach

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Clàr iontagrailean àbhaisteach

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

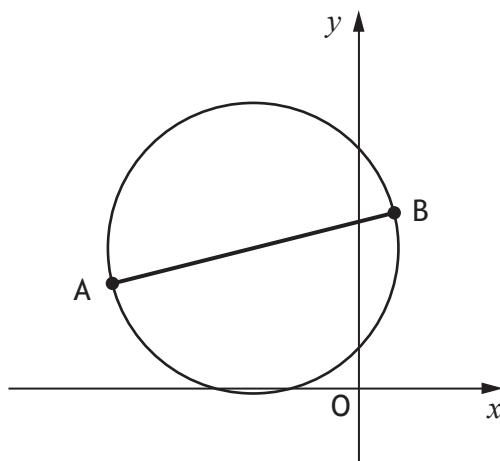
1. 'S e a th' ann an co-aontar an lùib $y = x^2 - 4x + 7$.

Lorg co-aontar an tadhaidh dhan lùb seo aig a' phuing far a bheil $x = 5$.

4

2. Tha A agus B nam puingean $(-7, 3)$ agus $(1, 5)$.

Is e AB trast-thomhas a' chearcaill.



Lorg co-aontar a' chearcaill seo.

3

3. 'S ann aig loidhne l_1 a tha an co-aontar $\sqrt{3}y - x = 0$.

(a) Tha loidhne l_2 ceart-cheàrnach ris l_1 . Lorg caisead l_2 .

2

(b) Obraich a-mach an ceàrn a tha l_2 a' dèanamh le càrsa dearbhte an x -axis.

2

4. Tha na puingean A(0, 9, 7), B(5, -1, 2), C(4, 1, 3) agus D(x , -2, 2) cho mòr is gu bheil \overrightarrow{AB} ceart-cheàrnach ri \overrightarrow{CD} .
 Obraich a-mach luach x .

4

5. Airson am polynominal, $x^3 - 4x^2 + ax + b$

- tha $x - 1$ na fhardtar
 - 's e - 12 an còrr an uair a tha e air an roinn le $x - 2$
- (a) Obraich a-mach luachan a agus b .
- (b) Mar sin, fuasgail $x^3 - 4x^2 + ax + b = 0$.

5

3

6. Tha sreath air a chruthachadh leis an dàimh ath-chuaireachaidh $u_{n+1} = mu_n + 6$ far a bheil m seasmhach.

- (a) Leis gu bheil $u_1 = 28$ agus $u_2 = 13$, lorg luach m .
- (b) (i) Mìnich carson a tha an t-sreath seo a' dlùthachadh ri crìoch mar $n \rightarrow \infty$.
 (ii) Obraich a-mach an crìoch seo.

2

1

2

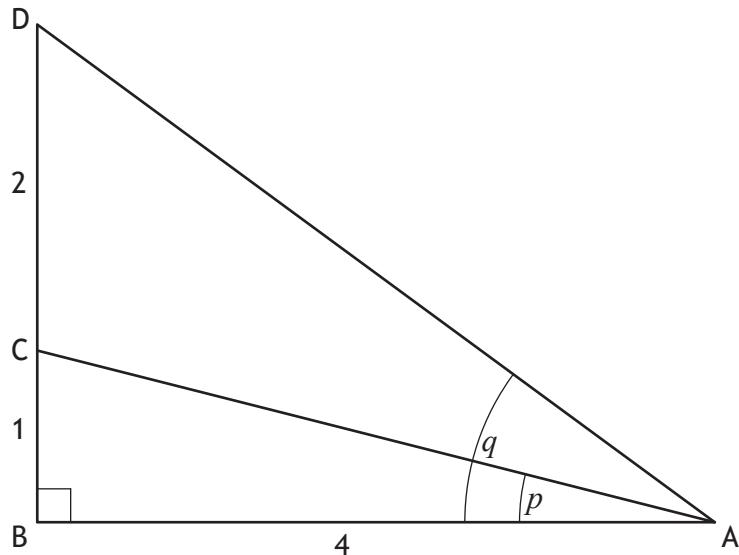
7. (a) Obraich a-mach luach $\log_5 25$.

1

- (b) Agus mar sin fuasgail $\log_4 x + \log_4 (x - 6) = \log_5 25$, far a bheil $x > 6$.

5

8. Tha triantan ABD ceart-cheàrnach aig B le ceàrnan $BAC = p$ agus $BAD = q$ agus faid mar a chithear san dealbh gu h-ìosal.



Seall gur e 19 17 an dearbh luach a th' aig $\cos(q - p) \frac{19\sqrt{17}}{85}$.

5

9. Tha an lùb $y = f(x)$ mar $\sin \frac{dy}{dx} = 4x - 6x^2$. Tha an lùb a' dol tron phuing $(-1, 9)$.
Sgrìobh y a rèir x.

4

10. (a) Fuasgail $\cos 2x^\circ - 3 \cos x^\circ + 2 = 0$ airson $0 \leq x < 360$.

5

- (b) Mar sin, fuasgail $\cos 4x^\circ - 3 \cos 2x^\circ + 2 = 0$ airson $0 \leq x < 360$.

2

[Tionndaidh an duilleag

11. Tha fuincsean f agus g gan innse air an seata fhìor-àireamhan le $f(x) = x^3 - 1$ agus $g(x) = 3x + 1$.

- (a) Lorg abairt airson $k(x)$, far a bheil $k(x) = g(f(x))$. 2
- (b) Ma $h(k(x)) = x$, lorg abairt airson $h(x)$. 3

[CRIODH A' PHÀIPEIR EISIMPLEIR]



National
Qualifications
SPECIMEN ONLY

S847/76/11

**Mathematics
Paper 1 (Non-calculator)**

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

The information in this publication may be reproduced to support SQA qualifications only on a non-commercial basis. If it is reproduced, SQA should be clearly acknowledged as the source. If it is to be used for any other purpose, written permission must be obtained from permissions@sqa.org.uk.

Where the publication includes materials from sources other than SQA (ie secondary copyright), this material should only be reproduced for the purposes of examination or assessment. If it needs to be reproduced for any other purpose it is the user's responsibility to obtain the necessary copyright clearance.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme – this indicates why each mark is awarded*
- *illustrative scheme – this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

- (a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.
- (b) If a candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your team leader.
- (c) One mark is available for each •. There are no half marks.
- (d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.
- (e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.
- (f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.
- (g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

- (h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x - 4x + 3 = 0$$

$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$

$$x - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 1 \text{ or } 3$$

(i) **Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

$$\begin{array}{ll} \bullet^5 & \bullet^6 \\ \bullet^5 \quad x = 2 & x = -4 \\ \bullet^6 \quad y = 5 & y = -7 \end{array}$$

Horizontal: $\bullet^5 x = 2$ and $x = -4$
 $\bullet^6 y = 5$ and $y = -7$

Vertical: $\bullet^5 x = 2$ and $y = 5$
 $\bullet^6 x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

- (j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4}{5}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

- (k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:
- working subsequent to a correct answer
 - correct working in the wrong part of a question
 - legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
 - omission of units
 - bad form (bad form only becomes bad form if subsequent working is correct), for example

$(x^3 + 2x^2 + 3x + 2)(2x + 1)$ written as

$$(x^3 + 2x^2 + 3x + 2) \times 2x + 1$$

$$= 2x^4 + 5x^3 + 8x^2 + 7x + 2$$

gains full credit

- repeated error within a question, but not between questions or papers

- (l) In any ‘Show that...’ question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.
- (m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate’s response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.
- (n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.
- (o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			<ul style="list-style-type: none"> •¹ differentiate •² calculate gradient •³ find the value of y •⁴ find equation of tangent 	<ul style="list-style-type: none"> •¹ $2x - 4$ •² 6 •³ 12 •⁴ $y = 6x - 18$ 	4
2.			<ul style="list-style-type: none"> •¹ find the centre •² calculate the radius •³ state equation of circle 	<ul style="list-style-type: none"> •¹ $(-3, 4)$ •² $\sqrt{17}$ •³ $(x+3)^2 + (y-4)^2 = 17$ or equivalent 	3
3.	(a)		<ul style="list-style-type: none"> •¹ find gradient l_1 •² state gradient l_2 	<ul style="list-style-type: none"> •¹ $\frac{1}{\sqrt{3}}$ •² $-\sqrt{3}$ 	2
	(b)		<ul style="list-style-type: none"> •³ using $m = \tan \theta$ •⁴ calculating angle 	<ul style="list-style-type: none"> •³ $\tan \theta = -\sqrt{3}$ •⁴ $\theta = \frac{2\pi}{3}$ or 120° 	2

Question		Generic scheme	Illustrative scheme	Max mark
4.		<ul style="list-style-type: none"> •¹ find \vec{CD} •² find \vec{AB} •³ equate scalar product to zero •⁴ calculate value of x 	<ul style="list-style-type: none"> •¹ $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$ •² $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$ •³ $5(x-4) + (-10)(-3) + (-5)(-1) = 0$ •⁴ $x = -3$ 	4

Question		Generic scheme	Illustrative scheme	Max mark	
5.	(a)	<ul style="list-style-type: none"> •¹ use appropriate strategy •² obtain an expression for a and b •³ obtain a second expression for a and b •⁴ find the value of a or b •⁵ find the second value 	<ul style="list-style-type: none"> •¹ $(1)^3 - 4(1)^2 + a(1) + b = 0$ •² $a + b = 3$ •³ $2a + b = -4$ •⁴ $a = -7$ or $b = 10$ •⁵ $b = 10$ or $a = -7$ 	5	
	(b)	<ul style="list-style-type: none"> •⁶ obtain quadratic factor •⁷ complete factorisation •⁸ state solutions 	<ul style="list-style-type: none"> •⁶ $(x^2 - 3x - 10)$ •⁷ $(x-1)(x-5)(x+2)$ •⁸ $x = 1, x = 5, x = -2$ 	3	
6.	(a)	<ul style="list-style-type: none"> •¹ interpret information •² solve to find m 	<ul style="list-style-type: none"> •¹ $13 = 28m + 6$ •² $m = \frac{1}{4}$ 	2	
	(b)	(i)	<ul style="list-style-type: none"> •³ state condition 	<ul style="list-style-type: none"> •³ a limit exists as $-1 < \frac{1}{4} < 1$ 	1
		(ii)	<ul style="list-style-type: none"> •⁴ know how to calculate limit •⁵ calculate limit 	<ul style="list-style-type: none"> •⁴ $L = \frac{1}{4}L + 6$ •⁵ $L = 8$ 	2

Question		Generic scheme	Illustrative scheme	Max mark
7.	(a)	• ¹ state value	• ¹ 2	1
	(b)	<ul style="list-style-type: none"> •¹ use laws of logarithms •² link to part (a) •³ use laws of logarithms •⁴ write in standard quadratic form •⁵ solve for x and identify appropriate solution 	<ul style="list-style-type: none"> •¹ $\log_4 x(x-6)$ •² $\log_4 x(x-6) = 2$ •³ $x(x-6) = 4^2$ •⁴ $x^2 - 6x - 16 = 0$ •⁵ 8 	5
8.		<ul style="list-style-type: none"> •¹ calculate lengths AC and AD •² select appropriate formula and express in terms of p and q •³ calculate two of $\cos p, \cos q, \sin p, \sin q$ •⁴ calculate other two and substitute into formula •⁵ arrange into required form 	<ul style="list-style-type: none"> •¹ $AC = \sqrt{17}$ and $AD = 5$ stated or implied by •³ •² $\cos q \cos p + \sin q \sin p$ stated or implied by •⁴ •³ $\cos p = \frac{4}{\sqrt{17}}$, $\cos q = \frac{4}{5}$ $\sin p = \frac{1}{\sqrt{17}}$, $\sin q = \frac{3}{5}$ •⁴ $\frac{4}{5} \times \frac{4}{\sqrt{17}} + \frac{3}{5} \times \frac{1}{\sqrt{17}}$ •⁵ $\frac{19}{5\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}} = \frac{19\sqrt{17}}{85}$ or $\frac{19}{5\sqrt{17}} = \frac{19\sqrt{17}}{5 \times 17} = \frac{19\sqrt{17}}{85}$ 	5

Question		Generic scheme	Illustrative scheme	Max mark
9.		<ul style="list-style-type: none"> •¹ know to and start to integrate •² complete integration •³ substitute for x and y •⁴ state expression for y 	<ul style="list-style-type: none"> •¹ eg $y = \frac{4}{2}x^2 \dots$ •² $y = \frac{4}{2}x^2 - \frac{6}{3}x^3 + c$ •³ $9 = 2(-1)^2 - 2(-1)^3 + c$ •⁴ $y = 2x^2 - 2x^3 + 5$ 	4
10.	(a)	<ul style="list-style-type: none"> •¹ use double angle formula •² express as a quadratic in $\cos x^\circ$ •³ start to solve •⁴ reduce to equations in $\cos x^\circ$ only •⁵ process solutions 	<p>Method 1: Using factorisation</p> <ul style="list-style-type: none"> •¹ $2\cos^2 x^\circ - 1 \dots$ stated or implied by •² •² $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ •³ $(2\cos x^\circ - 1)(\cos x^\circ - 1)$ <p>Method 2: Using quadratic formula</p> <ul style="list-style-type: none"> •¹ $2\cos^2 x^\circ - 1 \dots$ stated or implied by •² •² $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ stated explicitly •³ $\frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2}$ <p>In both methods:</p> <ul style="list-style-type: none"> •⁴ $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$ •⁵ $0, 60, 300$ or •⁴ $\cos x = 1$ and $x = 0$ •⁵ $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300 	5
	(b)	<ul style="list-style-type: none"> •⁶ interpret relationship with (a) •⁷ state valid values 	<ul style="list-style-type: none"> •⁶ $2x = 0$ and 60 and 300 •⁷ $0, 30, 150, 180, 210$ and 330 	2

Question		Generic scheme	Illustrative scheme	Max mark
11.	(a)	<ul style="list-style-type: none"> •¹ interpret notation •² complete process 	<ul style="list-style-type: none"> •¹ $g(x^3 - 1)$ •² $3x^3 - 2$ 	2
	(b)	<ul style="list-style-type: none"> •³ start to rearrange for x •⁴ rearrange •⁵ state expression for $h(x)$ 	<ul style="list-style-type: none"> •³ $3x^3 = y + 2$ •⁴ $x = \sqrt[3]{\frac{y+2}{3}}$ •⁵ $h(x) = \sqrt[3]{\frac{x+2}{3}}$ 	3

[END OF SPECIMEN MARKING INSTRUCTIONS]