National
Qualifications
2024

2024 Mathematics

Higher - Paper 2

Question Paper Finalised Marking Instructions

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$x=1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2+5x+7=9x+4$$
$$x-4x+3=0$$
$$(x-3)(x-1)=0$$
$$x=1 \text{ or } 3$$

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$
$\bullet^6$ $y=5$ and $y=-7$

Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$. or $1\frac{1}{4}$ $\quad$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\quad$ $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Question			Generic scheme	Illustrative scheme	Max mark
1.	(a)		•[1] determine midpoint of AC	•[1] (4,4)	3
			•[2] determine gradient of median	•[2] 2 or $\frac{10}{5}$	
			•[3] find equation of median	•[3] $y=2x-4$	

Notes:

1. •[2] is only available to candidates who use a midpoint to find a gradient.
2. •[3] is only available as a consequence of using a 'midpoint' of AC and the point B
3. At •[3] accept any arrangement of a candidate's equation where the constant terms have been simplified.
4. •[3] is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate A - perpendicular bisector of AC Midpoint = (4,4) •[1] ✓ $m_{AC}=-\frac{4}{7}\Rightarrow m_{\perp}=\frac{7}{4}$ •[2] ✗ $4y=7x-12$ •[3] $\checkmark_2$ For other perpendicular bisectors award 0/3	**Candidate B - altitude through B** $m_{AC}=-\frac{4}{7}$ •[1] ^ $m_{\perp}=\frac{7}{4}$ •[2] ✗ $4y=7x-17$ •[3] $\checkmark_2$
Candidate C - median through A midpoint BC = $(5,-3)$ •[1] ✗ $m_{AM}=-\frac{11}{8}$ •[2] $\checkmark_1$ $8y=-11x+31$ •[3] $\checkmark_2$	**Candidate D - median through C** midpoint AB $(-2,1)$ •[1] ✗ $m_{CM}=-\frac{1}{13}$ •[2] $\checkmark_1$ $13y=-x+11$ •[3] $\checkmark_2$

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[4] determine gradient of BC	•[4] $\frac{6}{12}$	3
			•[5] determine gradient of L	•[5] $-\frac{12}{6}$	
			•[6] find equation of L	•[6] $y=-2x+22$	

Notes:

5. •[6] is only available as a consequence of using a perpendicular gradient and C.
6. At •[6] accept any arrangement of a candidate's equation where the constant terms have been simplified.

Commonly Observed Responses:

Candidate E - altitude through C

$m_{AB}=-7$ •[4] ✗

$m_{\perp}=\frac{1}{7}$ •[5] $\checkmark_1$

$y=\frac{1}{7}(x-11)$ •[6] $\checkmark_1$

Question			Generic scheme	Illustrative scheme	Max mark
1.	(c)		•[7] determine x-coordinate •[8] determine y-coordinate	•[7] 6.5 or $\frac{13}{2}$ •[8] 9	**2**

Notes:

7. For $\left(\frac{26}{4}, 9\right)$ award 1/2.

Commonly Observed Responses:

Candidate F - rounding decimals

(a) $4y = 5x - 19$

(b) $y = -2x + 22$

(c) $x = \frac{107}{13} = 8.2$ •[7] ✓[1]

$y = 5.6$ •[8] ✓[1]

Question			Generic scheme	Illustrative scheme	Max mark
2.			•1 find y-coordinate •2 write in differentiable form •3 differentiate •4 find gradient of tangent •5 determine equation of tangent	•1 1 •2 $8x^{-3}$ •3 $8\times(-3)x^{-4}$ •4 $-\frac{3}{2}$ •5 $3x+2y=8$	5

Notes:

1. Only •1 and •2 are available to candidates who integrate. However, see Candidates E and F.
2. $8\times(-3)x^{-4}$ without previous working gains •2 and •3.
3. •3 is only available for differentiating a negative power. •4 and •5 are still available.
4. •4 is not available for $y=-\frac{3}{2}$. However, where $-\frac{3}{2}$ is then used as the gradient of the straight line, •4 may be awarded - see Candidates A, B and C.
5. •5 is only available where candidates attempt to find the gradient by substituting into their derivative.
6. •5 is not available as a consequence of using a perpendicular gradient.
7. Where $x=2$ has not been used to determine the y-coordinate, •5 is not available.

Commonly Observed Responses:

Candidate A - incorrect notation $y=1$ •1 ✓ - BoD $y=8x^{-3}$ •2 ✓ $y=-24x^{-4}$ •3 ✓ $y=-\frac{3}{2}$ •4 ✓ - BoD $3x+2y=8$ •5 ✓	**Candidate B - use of values in equation** $y=1$ •1 ✓ - BoD $y=8x^{-3}$ •2 ✓ $\frac{dy}{dx}=8\times(-3)x^{-4}$ •3 ✓ $\frac{dy}{dx}=-\frac{3}{2}$ •4 ✓ $y=-\frac{3}{2}$ $3x+2y=8$ •5 ✓
Candidate C - incorrect notation $y=1$ •1 ✓ - BoD $y=8x^{-3}$ •2 ✓ $\frac{dy}{dx}=8\times(-3)x^{-4}$ •3 ✓ $y=-\frac{3}{2}$ •4 ✗ **Evidence for •4 would need to appear in the equation of the line**	**Candidate D** $y=1$ •1 ✓ $y=8x^{-3}$ •2 ✓ $\frac{dy}{dx}=8\times(-3)x^{-4}=0$ •3 ✓ $8\times(-3)(2)^{-4}=0$ $m=-\frac{3}{2}$ •4 ✗ $3x+2y=8$ •5 ✓1

Question	Generic scheme	Illustrative scheme	Max mark
2. (continued)			

Candidate E - integrating in part		Candidate F - appearance of $+c$	
$y=1$	$\bullet^1$ ✓	$y=1$	$\bullet^1$ ✓
$y=8x^{-3}$	$\bullet^2$ ✓	$y=8x^{-3}$	$\bullet^2$ ✓
$\frac{dy}{dx}=-24x^{-2}$	$\bullet^3$ ✗	$\frac{dy}{dx}=-24x^{-4}+c$	$\bullet^3$ ✗ $\bullet^4$ ✗
$\frac{dy}{dx}=-6$	$\bullet^4$ $✓_1$		$\bullet^5$ ✗
$y=-6x+13$	$\bullet^5$ $✓_1$		

Question			Generic scheme	Illustrative scheme	Max mark
3.	(a)		•¹ find $\overrightarrow{ED}$	•¹ $\begin{pmatrix}1\\-4\\6\end{pmatrix}$	**2**
			•² find $\overrightarrow{EF}$	•² $\begin{pmatrix}2\\1\\3\end{pmatrix}$	

Notes:

1. For candidates who find **both** $\overrightarrow{DE}$ **and** $\overrightarrow{FE}$ correctly, award 1/2.
2. Accept vectors written horizontally.

Commonly Observed Responses:

	(b)	(i)	•³ evaluate $\overrightarrow{ED}.\overrightarrow{EF}$	•³ 16	**1**
		(ii)	•⁴ evaluate $\lvert\overrightarrow{ED}\rvert$	•⁴ $\sqrt{53}$	**4**
			•⁵ evaluate $\lvert\overrightarrow{EF}\rvert$	•⁵ $\sqrt{14}$	
			•⁶ substitute into formula for scalar product	•⁶ $\cos DEF = \frac{16}{\sqrt{53}\times\sqrt{14}}$ or $\sqrt{53}\times\sqrt{14}\times\cos DEF = 16$	
			•⁷ calculate angle	•⁷ 54.028…° or 0.942… radians	

Question	Generic scheme	Illustrative scheme	Max mark
3. (b) (continued)			

Notes:

3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating a magnitude. For example accept $\sqrt{1^2+4^2+6^2}=\sqrt{53}$ or $\sqrt{1^2+-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
   However, do not accept $\sqrt{1^2-4^2+6^2}=\sqrt{53}$ for $\bullet^4$.
4. $\bullet^6$ is not available to candidates who simply state the formula $\cos\theta=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{\left|\overrightarrow{ED}\right|\left|\overrightarrow{EF}\right|}$.
   However, $\cos\theta=\dfrac{16}{\sqrt{53}\times\sqrt{14}}$ and $\sqrt{53}\times\sqrt{14}\times\cos\theta=16$ are acceptable for $\bullet^6$.
5. Accept correct answers rounded to 54° or 0.9 radians (or 60 gradians).
6. Do not penalise the omission or incorrect use of units.
7. $\bullet^7$ is only available as a result of using a valid strategy.
8. $\bullet^7$ is only available for a single angle.
9. For a correct answer with no working award 0/4.

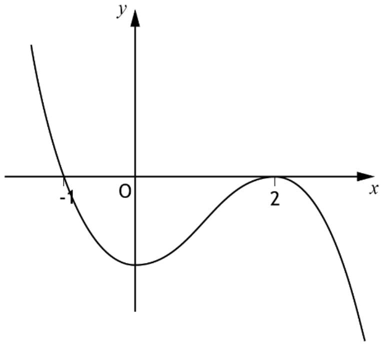
Commonly Observed Responses:

Candidate A - poor notation	Candidate B - insufficient communication
$\begin{pmatrix}1\\-4\\6\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix}=\begin{pmatrix}2\\-4\\18\end{pmatrix}=16$ $\quad\bullet^3$ ✗	$\left\|\overrightarrow{ED}\right\|=\sqrt{53}$ $\quad\bullet^4$ ✓ $\left\|\overrightarrow{EF}\right\|=\sqrt{14}$ $\quad\bullet^5$ ✓ $\dfrac{16}{\sqrt{53}\times\sqrt{14}}$ $\quad\bullet^6$ ^ 54.028…° or 0.942… radians $\quad\bullet^7$ ✓$_1$
Candidate C - BEWARE $\left\|\overrightarrow{OF}\right\|=\sqrt{14}$ $\quad\bullet^5$ ✗	

Question			Generic scheme	Illustrative scheme	Max mark
4.	(a)		•¹ identify x-coordinate •² identify y-coordinate	•¹ 3 •² 5	2

Notes:

Commonly Observed Responses:

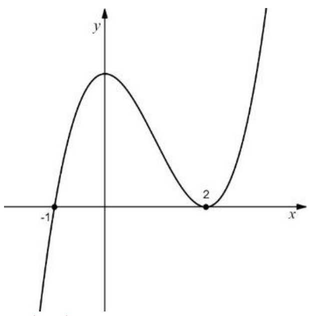
Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ identify roots •⁴ interpret point of inflection •⁵ identify orientation and complete cubic curve	•³ "cubic" with roots at −1 and 2 •⁴ "cubic" with turning point at (2,0) •⁵ cubic with maximum turning point at (2,0) 	3

Notes:

1. Note that the position of the minimum turning point of $f'(x)$ is not being assessed.
2. Where a candidate has not drawn a cubic curve or their graph does not extend outwith $-1 \le x \le 2$ award 0/3. However see Candidate D.
3. Do not penalise the appearance of an additional root outwith $-1 \le x \le 2$ (on a cubic curve) at •³.

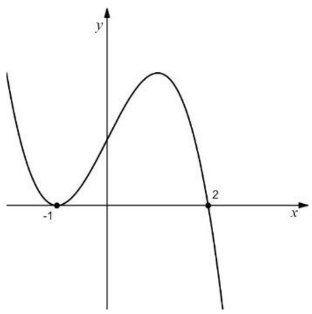
Commonly Observed Responses:

Candidate A - $-f'(x)$



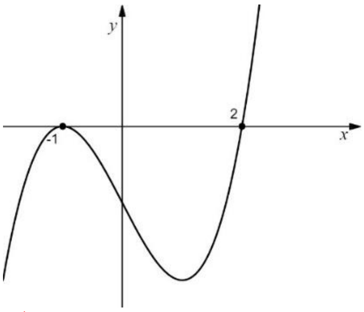
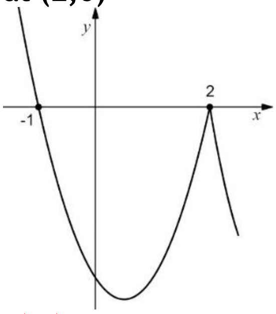
✓ ✓ ×

Candidate B



✓ × ×

Question	Generic scheme	Illustrative scheme	Max mark
4. (b) (continued)			

Candidate C  ✓ × ×	**Candidate D - left derivative ≠ right derivative at (2,0)**  ✓ ✓ ×

Question			Generic scheme	Illustrative scheme	Max mark
5.			•¹ integrate •² substitute limits •³ evaluate integral	•¹ $-\frac{1}{5}\cos 5x$ •² $\left(-\frac{1}{5}\cos\left(5\times\frac{\pi}{7}\right)\right)-\left(-\frac{1}{5}\cos(5\times 0)\right)$ •³ 0.3246...	3

Notes:

1. For candidates who differentiate throughout, make no attempt to integrate, or use another invalid approach (for example $\cos 5x^2$) award 0/3.
2. Do not penalise the inclusion of '$+c$' or the continued appearance of the integral sign after integrating.
3. Accept $\left(-\frac{1}{5}\cos 5\left(\frac{\pi}{7}\right)\right)-\left(-\frac{1}{5}\cos 5(0)\right)$ for •².
4. •³ is only available where candidates have considered both limits within a trigonometric function.

Commonly Observed Responses:

Candidate A - integrated in part $-\cos 5x$ •¹ ✗ $-\cos\left(\frac{5\pi}{7}\right)-\left(-\cos(5\times 0)\right)$ •² $\checkmark_1$ 1.623... •³ $\checkmark_1$	**Candidate B - insufficient evidence of integration** $\cos 5x$ •¹ ✗ $\cos\left(\frac{5\pi}{7}\right)-\left(\cos(5\times 0)\right)$ •² $\checkmark_2$ −1.623 •³ $\checkmark_2$
Candidate C - insufficient evidence of integration $\frac{1}{5}\sin 5x$ •¹ ✗ $\frac{1}{5}\sin\frac{5\pi}{7}-\frac{1}{5}\sin 0$ •² $\checkmark_2$ 0.156... •³ $\checkmark_2$	**Candidate D - working in degrees before integrating** $\int_0^{25.7...}\sin 5x\,dx$ •¹ ✗ $-\frac{1}{5}\cos 5x$ $\left(-\frac{1}{5}\cos 128.57...\right)-\left(-\frac{1}{5}\cos 0\right)$ •² $\checkmark_1$ 0.3246... •³ $\checkmark_1$

Question			Generic scheme	Illustrative scheme	Max mark
6.			**Method 1**	**Method 1**	5
			•¹ state linear equation	•¹ $\log_5 y = 3\log_5 x - 2$	
			•² introduce logs	•² $\log_5 y = 3\log_5 x - 2\log_5 5$	
			•³ use laws of logs	•³ $\log_5 y = \log_5 x^3 - \log_5 5^2$	
			•⁴ use laws of logs	•⁴ $\log_5 y = \log_5 \frac{x^3}{5^2}$	
			•⁵ state a and b	•⁵ $a = \frac{1}{25}, b = 3$ or $y = \frac{x^3}{25}$	
			Method 2	**Method 2**	5
			•¹ state linear equation	•¹ $\log_5 y = 3\log_5 x - 2$	
			•² use laws of logs	•² $\log_5 y = \log_5 x^3 - 2$	
			•³ use laws of logs	•³ $\log_5 \frac{y}{x^3} = -2$	
			•⁴ use laws of logs	•⁴ $\frac{y}{x^3} = 5^{-2}$	
			•⁵ state a and b	•⁵ $a = \frac{1}{25}, b = 3$ or $y = \frac{x^3}{25}$	
			Method 3	**Method 3** **The equations at •¹, •² and •³ must be stated explicitly**	5
			•¹ introduce logs to $y = ax^b$	•¹ $\log_5 y = \log_5 ax^b$	
			•² use laws of logs	•² $\log_5 y = b\log_5 x + \log_5 a$	
			•³ interpret intercept	•³ $\log_5 a = -2$	
			•⁴ use laws of logs	•⁴ $a = \frac{1}{25}$	
			•⁵ interpret gradient	•⁵ $b = 3$	

Question	Generic scheme	Illustrative scheme	Max mark

6. (continued)

Notes

1. In any method, marks may only be awarded within a valid strategy using $y = ax^b$. For example, see Candidates C and D.
2. Markers must identify the method which best matches the candidate's approach; markers must not mix and match between methods.
3. Penalise the omission of base 5 at most once in any method.
4. Where candidates use an incorrect base then only $\bullet^2$ **and** $\bullet^3$ are available (in any method).
5. Do not accept $a = 5^{-2}$.
6. In Method 3, do not accept $m = 3$ or gradient $= 3$ for $\bullet^5$.
7. Do not penalise candidates who score out "log" from equations of the form $\log_5 m = \log_5 n$.

Commonly Observed Responses

Candidate A - missing equations at $\bullet^1$, $\bullet^2$ and $\bullet^3$ in Method 3

$a = \frac{1}{25}$ $\bullet^4$ ✓

$b = 3$ $\bullet^5$ ✓

Candidate B - no working - Method 3

$b = \frac{1}{25}$ $\bullet^4$ ✗

$a = 3$ $\bullet^5$ ✗

Candidate C - Method 2

$y = 3x - 2$

$\log_5 y = 3\log_5 x - 2$ $\bullet^1$ ✓

$\log_5 y = \log_5 x^3 - 2$ $\bullet^2$ ✓

$y = x^3 - 2$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗

Candidate D - Method 2

$\log_5 y = 3\log_5 x - 2$ $\bullet^1$ ✓

$\log_5 y = \log_5 x^3 - 2$ $\bullet^2$ ✓

$\frac{y}{x^3} = -2$ $\bullet^3$ ✗ $\bullet^4$ ✗ $\bullet^5$ ✗

Candidate E - use of coordinate pairs

$\log_5 x = 4$ **and** $\log_5 y = 10$ $\bullet^1$ ✓

$x = 5^4$ and $y = 5^{10}$ $\bullet^2$ ✓

$\log_5 x = 0$, $\log_5 y = -2$

$\Rightarrow x = 1,\ y = 5^{-2}$ $\bullet^3$ ✓

$5^{-2} = a \times 1^b \Rightarrow a = \frac{1}{25}$ $\bullet^4$ ✓

$5^{10} = 5^{-2} \times 5^{4b} \Rightarrow -2 + 4b = 10$

$\Rightarrow b = 3$ $\bullet^5$ ✓

Candidates may use $(0,-2)$ for $\bullet^1$ and $\bullet^2$ and $(4,10)$ for $\bullet^3$.

Question			Generic scheme	Illustrative scheme	Max mark
7.			**Method 1**	**Method 1**	**5**
			•¹ integrate using 'upper' – 'lower'	•¹ $\int\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•² identify limits	•² $\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$	
			•³ integrate	•³ $6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4$	
			•⁴ substitute limits	•⁴ $\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right)-0$	
			•⁵ evaluate area	•⁵ $\frac{14}{3}$ (units²)	
			Method 2	**Method 2**	
			•¹ know to integrate between appropriate limits for both equations	•¹ $\int_0^2 \ldots dx$ and $\int_0^2 \ldots dx$	
			•² integrate both functions	•² $6x+\frac{4x^2}{2}-\frac{2x^3}{3}$ and $\frac{x^4}{4}-\frac{6x^3}{3}+\frac{11x^2}{2}$	
			•³ substitute limits into both expressions	•³ $\left(6(2)+\frac{4(2)^2}{2}-\frac{2(2)^3}{3}\right)-0$ and $\left(\frac{(2)^4}{4}-\frac{6(2)^3}{3}+\frac{11(2)^2}{2}\right)-0$	
			•⁴ evaluate both integrals	•⁴ $\frac{44}{3}$ and 10	
			•⁵ evidence of subtracting areas	•⁵ $\frac{14}{3}$ (units²)	

Question	Generic scheme	Illustrative scheme	Max mark

7. (continued)

Notes:

1. Correct answer with no working - award 1/5.
2. Do not penalise lack of 'dx' at $\bullet^1$ in Method 1.
3. In Method 1, limits and 'dx' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded and in Method 2 by the $\bullet^1$ stage for $\bullet^1$ to be awarded.
4. In Method 1, treat the absence of brackets at $\bullet^1$ stage as bad form only if the correct integrand is obtained. See Candidates C and D.
5. Where a candidate differentiates one or more terms, or fails to integrate, no further marks are available.
6. In Method 1, accept unsimplified expressions such as $6x+\frac{4x^2}{2}-\frac{2x^3}{3}-\frac{x^4}{4}+\frac{6x^3}{3}-\frac{11x^2}{2}$ at $\bullet^3$.
7. Do not penalise the inclusion of '$+c$'.
8. Do not penalise the continued appearance of the integral sign or dx after integrating.
9. $\bullet^5$ is not available where solutions include statements such as '$-\frac{14}{3}=\frac{14}{3}$ square units'. See Candidates A and B.
10. In Method 1, where a candidate uses an invalid strategy the only marks available are $\bullet^3$ for integrating a polynomial with at least four terms (of different degree) and $\bullet^4$ for substituting the limits of 0 and 2 into the resulting expression. However, see Candidate E.
11. At $\bullet^4$, do not penalise candidates for who reduce powers of 0. For example writing 0 in place of 0^4. Similarly, do not penalise candidates writing 0 in place of $6(0)$. However, candidates who write 0^3 in place of 0^4 or $2(0)$ in place of $6(0)$ do not gain $\bullet^4$.

Commonly Observed Responses:

Candidate A - switched limits

$\int_2^0 \left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$ $\bullet^2$ ✓

$=6x-\frac{7}{2}x^2+\frac{4}{3}x^3-\frac{1}{4}x^4$ $\bullet^3$ ✓

$=0-\left(6(2)-\frac{7}{2}(2)^2+\frac{4}{3}(2)^3-\frac{1}{4}(2)^4\right)$ $\bullet^4$ ✓

$=-\frac{14}{3}$

$=\frac{14}{3}$ $\bullet^1$ ✗ $\bullet^5$ ✗

Candidate B - 'lower' - 'upper'

$\int_0^2 \left(\left(x^3-6x^2+11x\right)-\left(6+4x-2x^2\right)\right)dx$ $\bullet^2$ ✓

$\int_0^2 x^3-4x^2+7x-6\,dx$

$=\frac{1}{4}x^4-\frac{4}{3}x^3+\frac{7}{2}x^2-6x$ $\bullet^3$ ✓

$\left(\frac{1}{4}(2)^4-\frac{4}{3}(2)^3+\frac{7}{2}(2)^2-6(2)\right)-(0)$ $\bullet^4$ ✓

$=-\frac{14}{3}$

$\therefore$ Area $=\frac{14}{3}$ $\bullet^1$ ✓ $\bullet^5$ ✓

Question	Generic scheme	Illustrative scheme	Max mark
7. (continued)			

Candidate C - missing brackets

$$\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$$

$$\int_0^2 6-7x+4x^2-x^3\,dx$$ $\bullet^1$ ✓ $\bullet^2$ ✓

Candidate D - missing brackets

$$\int_0^2 6+4x-2x^2-x^3-6x^2+11x\,dx$$ $\bullet^1$ × $\bullet^2$ ✓$_1$

$$\int_0^2 6+15x-8x^2-x^3\,dx$$

$$6x+\frac{15}{2}x^2-\frac{8}{3}x^3-\frac{1}{4}x^4$$ $\bullet^3$ ✓$_1$

$$\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3-\frac{1}{4}(2)^4\right)-(0)$$ $\bullet^4$ ✓$_1$

$$\frac{50}{3}$$ $\bullet^5$ ✓$_1$

Candidate E - ‘upper’ + ‘lower’

$$\int_0^2\left(\left(6+4x-2x^2\right)+\left(x^3-6x^2+11x\right)\right)dx$$ $\bullet^1$ × $\bullet^2$ ✓$_1$

$$6x+\frac{15}{2}x^2-\frac{8}{3}x^3+\frac{1}{4}x^4$$ $\bullet^3$ ✓$_1$

$$\left(6(2)+\frac{15}{2}(2)^2-\frac{8}{3}(2)^3+\frac{1}{4}(2)^4\right)-0$$ $\bullet^4$ ✓$_1$

$$\frac{74}{3}$$ $\bullet^5$ ✓$_1$

Candidate F - incorrect substitution

$$\int_0^2\left(\left(6+4x-2x^2\right)-\left(x^3-6x^2+11x\right)\right)dx$$ $\bullet^1$ ✓ $\bullet^2$ ✓

$$\left(6x+2x^2-\frac{2}{3}x^3\right)-\left(\frac{1}{4}x^4-2x^3+\frac{11}{2}x^2\right)$$ $\bullet^3$ ✓

$$\left(6(2)+2(2)^2-\frac{2}{3}(2)^3\right)-\left(\frac{1}{4}(0)^4-2(0)^3+\frac{11}{2}(0)^2\right)$$ $\bullet^4$ ×

$$\frac{44}{3}$$ $\bullet^5$ ✓$_2$

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		$\bullet^1$ interpret notation $\bullet^2$ state expression for $f(g(x))$	$\bullet^1$ $f(x+1)$ or $2g(x)^2-18$ $\bullet^2$ $2(x+1)^2-18$	2

Notes:

1. For $2(x+1)^2-18$ without working, award both $\bullet^1$ and $\bullet^2$.

Commonly Observed Responses:

Candidate A - $g(f(x))$		Candidate B - beware of two "attempts"	
$2x^2-17$	$\bullet^1$ ✗ $\bullet^2$ $\checkmark_1$	$f(g(x))=2x^2-18$ $f(x+1)=2(x+1)^2-18$	$\bullet^1$ ✗ $\bullet^2$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^3$ apply condition $\bullet^4$ state values of x	$\bullet^3$ $2(x+1)^2-18=0$ $\bullet^4$ -4 and 2	2

Notes:

2. Working at $\bullet^3$ must be consistent with working at $\bullet^2$.
3. Accept $2(x+1)^2-18\neq 0$ for $\bullet^3$ only when $x=-4$ and $x=2$ are stated explicitly at $\bullet^4$. See Candidate H
4. $\bullet^4$ is only available for finding the roots of a quadratic.
5. For subsequent incorrect working, the final mark is not available. For example $-4<x<2$.

Commonly Observed Responses:

Candidate C - expanding brackets in (a)		Candidate D - expanding brackets in (a)	
Part (a)		**Part (a)**	
$f(g(x))=2(x+1)^2-18$	$\bullet^1$ ✓ $\bullet^2$ ✓	$f(g(x))=2(x+1)^2-18$	$\bullet^1$ ✓ $\bullet^2$ ✓
$f(g(x))=2x^2+4x-16$		$f(g(x))=2x^2-16$	
Part (b)		Part (b)	
$2x^2+4x-16=0$	$\bullet^3$ ✓	$2x^2-16=0$	$\bullet^3$ ✗
$x=-4$ and $x=2$	$\bullet^4$ ✓	$x=\pm\sqrt{8}$	$\bullet^4$ $\checkmark_1$

Candidate E - $g(f(x))$		Candidate F - equivalent condition	
Part (a)			
$f(g(x))=2x^2-17$	$\bullet^1$ ✗ $\bullet^2$ $\checkmark_1$	$2(x+1)^2=18$	$\bullet^3$ ✓
Part (b)			
$2x^2-17=0$	$\bullet^3$ $\checkmark_1$		
$x=\pm\sqrt{\dfrac{17}{2}}$	$\bullet^4$ $\checkmark_1$		

Candidate G - use of $\neq$		Candidate H - use of $\neq$	
$2(x+1)^2-18\neq 0$	$\bullet^3$ ✗	$2(x+1)^2-18\neq 0$	
$x\neq -4$, $x\neq 2$	$\bullet^4$ $\checkmark_1$	$x\neq -4$, $x\neq 2$	
		$x=-4$, $x=2$	$\bullet^3$ ✓
		$\bullet^4$ ✓	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•¹ differentiate two non-constant terms •² complete derivative and equate to 0 •³ find x-coordinates •⁴ find y-coordinates	•¹ eg $x^2 - 2x$ •² $x^2 - 2x - 3 = 0$ •³ •⁴ •³ -1, 3 •⁴ $\frac{8}{3}$, -8	4

Notes:

1. For a numerical approach, award 0/4.
2. •² is only available if '$=0$' appears at the •² stage or in working leading to •³. However, see Candidate A.
3. •³ is only available for solving a quadratic equation.
4. •³ and •⁴ may be awarded vertically.

Commonly Observed Responses:

Candidate A	**Candidate B - derivative never equated to 0**
Stationary points when $\frac{dy}{dx} = 0$ $\frac{dy}{dx} = x^2 - 2x - 3$ •¹ ✓ •² ✓ $\frac{dy}{dx} = (x+1)(x-3)$ $x = -1, 3$ •³ ✓ $y = \frac{8}{3}, -8$ •⁴ ✓	$x^2 - 2x - 3$ •¹ ✓ •² ^ $(x+1)(x-3)$ $x = -1, 3$ •³ ✓₁ $y = \frac{8}{3}, -8$ •⁴ ✓

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•⁵ evaluate y at $x = 6$ •⁶ state greatest and least values	•⁵ 19 •⁶ greatest = 19 and least = -8	2

Notes:

5. 'Greatest $(6, 19)$; least $(3, -8)$' does not gain •⁶.
6. Where $x = -1$ was not identified as a stationary point in part (a), y must also be evaluated at $x = -1$ to gain •⁶.
7. •⁶ is not available for using y at a value of x, obtained at •³ stage, which lies outwith the interval $-1 \le x \le 6$.
8. •⁶ is only available where candidates have attempted to evaluate y at $x = 6$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•¹ state centre •² calculate radius	•¹ $(-9,1)$ •² $\sqrt{90}$ or $3\sqrt{10}$ or 9.48…	**2**

Notes:

1. Accept $x=-9,\ y=1$ for •¹.
2. Do not accept '$g=-9,\ f=1$' or '$-9,1$' for •¹.
3. Do not penalise candidates who treat negative signs with a lack of rigour when calculating the radius. For example accept $\sqrt{9^2+-1^2+8}=\sqrt{90}$ or $\sqrt{9^2+1^2+8}=\sqrt{90}$ or $\sqrt{-9^2+1^2+8}=\sqrt{90}$ for •². However, do not accept $\sqrt{9^2-1^2+8}=\sqrt{90}$ for •².

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•³ determine the distance between the centres and subtract to find a numerical expression for the radius of C_2 •⁴ determine equation of C_2	•³ eg $\sqrt{90}-\sqrt{10}$ •⁴ $(x+6)^2+y^2=40$	**2**

Notes:

4. Do not penalise the use of decimals.
5. The distance between the centres, and the radius of C_2 must be consistent with the sizes of the circles in the original diagram ($d<r_{C_2}<r_{C_1}$).
6. Where a candidate uses an incorrect radius without supporting working, •⁴ is not available.

Commonly Observed Responses:

Candidate A - follow-through marking

Part (a)

$r=\sqrt{74}$ •² ✗

Part (b)

$d=\sqrt{10}$

radius $=\sqrt{74}-\sqrt{10}$ •³ ✓₁

$(x+6)^2+y^2=5.44\ldots^2$

$(x+6)^2+y^2=29.59\ldots$ (or $84-4\sqrt{185}$) •⁴ ✓₁

Candidate B - using line through centres

Equation of radius: $3y=-x-6$

$(-3y-6)^2+y^2+18(-3y-6)-2y-8=0$

$10y^2-20y-80=0$

$\cancel{y=4}$ $y=-2$

$\cancel{x=-18}$ $x=0$

Radius = distance between $(-6,0)$ and $(0,-2)$

Radius = $\sqrt{40}$ •³ ✓

$(x+6)^2+y^2=40$ •⁴ ✓

Question			Generic scheme	Illustrative scheme	Max mark
11.	(a)		•^{1} state number of vehicles	•^{1} 6.8 million	**1**

Notes:

1. Accept 6.8 or $N = 6.8$ million for •^{1}.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•^{2} substitute for N and t	•^{2} $125 = 6.8e^{10k}$ stated or implied by •^{3}	**4**
			•^{3} process equation	•^{3} $\dfrac{125}{6.8} = e^{10k}$	
			•^{4} express in logarithmic form	•^{4} $\log_e\left(\dfrac{125}{6.8}\right) = 10k$	
			•^{5} solve for k	•^{5} 0.2911...	

Notes:

2. Accept answers which round to 0.29.
3. Do not penalise rounding or transcription errors (which are correct to 2 significant figures) in intermediate calculations.
4. •^{3} may be assumed by •^{4}.
5. Any base may be used at •^{4} stage. See Candidate A.
6. At •^{4} all exponentials must be processed.
7. Accept $\log_e \dfrac{125}{6.8} = 10k \log_e e$ for •^{4}.
8. The calculation at •^{5} must follow from the valid use of exponentials and logarithms at •^{3} and •^{4}.
9. For candidates with no working, or who adopt an iterative approach to arrive at $k = 0.29$, award 1/4. However, if, in the iterations N is calculated for $k = 0.295$ and $k = 0.285$, then award 4/4.

Commonly Observed Responses:

Candidate A - use of alternative base

$125 = 6.8e^{10k}$ •^{2} ✓

$\dfrac{125}{6.8} = e^{10k}$ •^{3} ✓

$\log_{10}\left(\dfrac{125}{6.8}\right) = 10k \log_{10} e$ •^{4} ✓

$k = 0.2911...$ •^{5} ✓

Candidate B - missing lines of working

$125 = 6.8e^{10k}$ •^{2} ✓

$k = 0.2911...$ •^{3} ^ •^{4} ^ •^{5} ✓

Candidate C - errors in substitution

$125\,000\,000 = 6.8e^{10k}$ •^{2} ×

$\dfrac{125\,000\,000}{6.8} = e^{10k}$ •^{3} ✓$_1$

$16.726... = 10k$ •^{4} ✓$_1$

$k = 1.6726...$ •^{5} ✓$_1$

Question			Generic scheme	Illustrative scheme	Max mark
12.			$\bullet^1$ substitute appropriate double angle formula	$\bullet^1$ $2(2\sin x^\circ \cos x^\circ) - \sin^2 x^\circ (= 0)$	5
			$\bullet^2$ factorise	$\bullet^2$ $\sin x^\circ (4\cos x^\circ - \sin x^\circ) = 0$	
			$\bullet^3$ solve for $\tan x^\circ$	$\bullet^3$ $\tan x^\circ = 4$ (since $x = 90, 270$ are not solutions)	
				$\bullet^4$ $\bullet^5$	
			$\bullet^4$ solve $\tan x^\circ = 4$	$\bullet^4$ 76, 256	
			$\bullet^5$ solve $\sin x^\circ = 0$	$\bullet^5$ 0, 180	

Notes:

1. $\bullet^1$ is still available to candidates who correctly substitute for $\sin^2 x$ in addition to $\sin 2x$.
2. Substituting $2\sin A \cos A$ for $\sin 2x^\circ$ at the $\bullet^1$ stage should be treated as bad form provided the equation is written in terms of x at the $\bullet^2$ stage. Otherwise, $\bullet^1$ is not available.
3. '$= 0$' must appear by the $\bullet^2$ stage for $\bullet^2$ to be awarded.
4. Award $\bullet^2$ for '$S(4C - S) = 0$' only where $\sin x^\circ = 0$ and $4\cos x^\circ - \sin x^\circ = 0$ appear.
5. Do not penalise the omission of degree signs.
6. At $\bullet^3$ stage, candidates are not required to check that 90 and 270 are not solutions before dividing by $\cos x^\circ$. Where candidates have divided by $\sin x$ at the $\bullet^2$ stage without considering $\sin x = 0$, $\bullet^3$ and $\bullet^4$ are still available.
7. At $\bullet^3$ stage, candidates may use the wave function and arrive at $\sqrt{17}\cos(x + 14)^\circ = 0$, or an equivalent wave form, instead of $\tan x^\circ = 4$.
8. $\bullet^4$ is only available where the working at the $\bullet^3$ stage is of equivalent difficulty to reaching $\tan x^\circ = 4$.
9. $\bullet^5$ is not available where $\sin x = 0$ comes from an invalid strategy.
10. For candidates who work only in radians, $\bullet^5$ is not available.
11. $\bullet^4$ and $\bullet^5$ may be awarded vertically. See also Candidate B.
12. Do not penalise solutions outwith $0 \le x < 360$.

Commonly Observed Responses:

Candidate A - working in radians

$\vdots$ — $\bullet^1$ ✓ $\bullet^2$ ✓

$\tan x^\circ = 4$ — $\bullet^3$ ✓

1.326, 4.468 — $\bullet^4$ $\checkmark_1$

$0, \pi$ — $\bullet^5$ $\checkmark_2$

Candidate B - partial solutions

$2(2\sin x^\circ \cos x^\circ) - \sin^2 x^\circ = 0$ — $\bullet^1$ ✓

$\sin x^\circ (4\cos x^\circ - \sin x^\circ) = 0$ — $\bullet^2$ ✓

$\sin x^\circ = 0$

$x = 180$

$\tan x^\circ = 4$ — $\bullet^3$ ✓

$x = 76$ — $\bullet^4$ ✓

$\bullet^5$ ^

Question			Generic scheme	Illustrative scheme	Max mark
13.			•1 state repeated factor	•1 $(x-3)^2(\dots)(\dots)$	3
			•2 state non-repeated linear factors	•2 $(\dots)^2(x+1)(x-5)$	
			•3 calculate k and express in required form	•3 $f(x)=\frac{1}{5}(x-3)^2(x+1)(x-5)$	

Notes:

1. Do not penalise the omission of $f(x)=$ or the inclusion of $y=$.
2. Accept $f(x)=\frac{1}{5}(x+-3)^2(x+1)(x+-5)$ for •3.

Commonly Observed Responses:

Candidate A - incorrect signs		Candidate B - incorrect repeated root	
$f(x)=k(x+3)^2(x-1)(x+5)$	•1 ✗ •2 ✓1	$f(x)=k(x+1)^2(x-3)(x-5)$	•1 ✗ •2 ✓1
$f(x)=\frac{1}{5}(x+3)^2(x-1)(x+5)$	•3 ✓1	$f(x)=-\frac{3}{5}(x+1)^2(x-3)(x-5)$	•3 ✓1
Candidate C - incorrect repeated root		**Candidate D - incorrect signs and repeated root**	
$f(x)=k(x-5)^2(x+1)(x-3)$	•1 ✗ •2 ✓1	$f(x)=k(x+5)^2(x-1)(x+3)$	•1 ✗ •2 ✗
$f(x)=\frac{3}{25}(x-5)^2(x+1)(x-3)$	•3 ✓1	$f(x)=\frac{3}{25}(x+5)^2(x-1)(x+3)$	•3 ✓1
Candidate E - incorrect signs and repeated root		**Candidate F - use of a, b and c**	
$f(x)=k(x-1)^2(x+5)(x+3)$	•1 ✗ •2 ✗	$a=-3$	•1 ✓
		$b=1,\ c=-5$ (or $b=-5,\ c=1$)	•2 ✓
$f(x)=-\frac{3}{5}(x-1)^2(x+5)(x+3)$	•3 ✓1	$k=\frac{1}{5}$	•3 ^

[END OF MARKING INSTRUCTIONS]